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XV. *The Influence of Small Changes of Temperature on Atmospheric Refraction.* *By* Sir Arthur Schuster, *F.R.S.*

Received February 4, 1920.

1. It has been suggested that the changes of temperature caused by the moon's shadow sweeping through our atmosphere may have an appreciable effect on optical refraction, and hence on the apparent position of stars near the sun. I propose to examine the nature of the effect and its order of magnitude. The investigations will be sufficiently general to apply also to other cases where the air is affected by systematic changes of temperature.

The optical length of a ray of light passing through a gas depends mainly on the total mass of gas traversed. If μ denote the refractive index, and ρ the density $(\mu-1)/\rho$ remains sensibly constant, so that a small change $\delta\rho$ increases the refractive index by $(\mu-1)\delta\rho/\rho$. Inequalities of pressure at two adjacent points of a horizontal surface are quickly removed by convection currents. The differences between the refractive indices at two such points will, therefore, be mainly determined by the differences in temperature, and a small fall in temperature τ will be connected with a change $\delta\rho$ in density by the relation $\delta\rho/\rho=\tau/t$, the temperature being measured on the absolute scale. Along each horizontal surface we have

$$\delta\mu=(\mu-1)\tau/t=(\mu_0-1)\rho\tau/\rho_0 t, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where μ_0, ρ_0 are the values of μ, ρ at some standard temperature—*e.g.*, 0°C. For the change of density with altitude I shall adopt the exponential formula, so that at altitude y the density is $\rho e^{-h_1 y}$.

The variation of temperature along the surface of the ground in the direction of the path of the moon's shadow is represented with sufficient accuracy by

$$\tau=\tfrac{1}{2}\tau_0(1+\cos m(x+a)), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

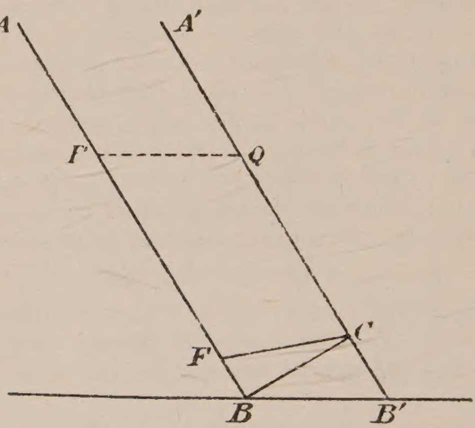
when x is measured from the centre of the umbra, and π/m is the radius of the penumbra, τ_0 denoting the maximum *fall* of temperature. The length a is introduced to allow for the displacement of the point of lowest temperature, which generally occurs at, or even after, the end of totality. The complete expression for τ would require additional terms involving multiples of mx; they might easily be taken into account if high accuracy were aimed at.

There is greater difficulty in estimating the influence of altitude on the changes of temperature. The calculation is simplified by introducing the factor $te^{-h_2 y}/t_0$ for the purpose, and by a proper choice of h_2 we may obtain an approximation to the variation which in any case will be much affected by local circumstance. We must finally provide for a possible change of phase, which may be taken to be proportional to the altitude. Irrelevant complications are avoided by treating the problem as two-dimensional—*i.e.*, by assuming the central line of the moon's shadow to pass through the centre of the earth.

Writing $h=h_1+h_2$, we then obtain

$$\rho\tau/\rho_0 t=\tfrac{1}{2}\tau_0 e^{-hy}(1+\cos . m(x+a-y\tan\theta)), \quad . \quad . \quad . \quad . \quad (3)$$

where θ is the angle between the vertical and the line joining the minima of temperature at different levels. This line may



for convenience be called the line of slope of temperature. Combining this with (1), we find

$$t_0\delta\mu=\tfrac{1}{2}(\mu_0-1)\tau_0 e^{-hy}(1+\cos . m(x+a-y\tan\theta)). \quad . \quad . \quad (4)$$

Let AB be a ray of light which will be slightly curved owing to the normal atmospheric refraction, and let $A'B'$ be an adjacent ray coming from the same distant source of light. The points A and A' are taken on the same horizontal lines at a sufficient elevation to allow us to disregard any change of refractive index. The wave-front BC is at right angles to AB. If the refractive index along PQ be raised from μ to $\mu+\delta\mu$ the excess of $\delta\mu$ at Q above that at P is $s\, d\delta\mu/dx$, s

being the distance PQ. The total increase of optical length of $A'B'$ over that of AB is

$$f=\int_0^\infty \frac{d\,.\,\delta\mu}{dx}\,dl,$$

where l is measured along AB. To correct for this increase, the wave-front is pushed back through a distance $BF=\mu_0 f$, where we may take the value of μ_0 to be essentially equal to unity. The angle ε through which the ray is turned is now determined by

$$-\tan\varepsilon=CF/BC=s^{-1}\sec\gamma\int_0^\infty \frac{d\delta\mu}{dx}\,dl,$$

where the negative sign indicates a diminution of the angle between the ray and the vertical; or, substituting (4)

$$\tan\varepsilon=Cm\tau_0\sec\gamma\int_0^\infty e^{-hy}\sin m(x+a-y\tan\theta). \quad . \quad . \quad (5)$$

C is a numerical factor equal to $(\mu_0-1)/2\mu_0\tau_0$, which for atmospheric air is $5{\cdot}4\times10^{-7}$. In deducing the deflection of a ray produced by small changes in the refractive index at different points, we have measured the changes of optical length along the original path of the ray; this is allowable so long as the altered path lies close to it. There is, therefore, here a tacit assumption that the deviation is small, and we may substitute ε for $\tan\varepsilon$, with the reservation, however, that the result will only be correct so long as ε^2 is negligible.

Before performing the integration we substitute

$$l=y\sec\gamma,\ x=x_0+y\tan\gamma.$$

Here x_0 denotes the distance of the point of observation from the centre of the umbra. Strictly speaking, the upper limit of the integral should not be infinite, because (2) holds only as far as $x=\pm\pi/m$, τ being zero for larger values of x; but considering the numerical value of h and m the error committed is negligible.

The expression for the integral is simplified by the following substitutions :—

$$c=x_0+a;\ \varphi=\gamma-\theta;\ n=m\sec\theta\sec\gamma;\ \tan\psi=n\sin\varphi/h.$$

We then obtain from (5)

$$\varepsilon=Cm\tau_0\sec^2\gamma\sin(mc+\psi)/(h^2+n^2\sin^2\varphi)^{\frac{1}{2}}. \quad . \quad . \quad (6)$$

This equation contains the solution of our problem. If we disregard the diminution of density with altitude, which is

equivalent to assuming zero value for h, we obtain Prof. Anderson's result ("Nature," December 4, 1919).* In that case ψ is a right angle, and at the point of minimum temperature ($c=o$) the deflection ε is inversely proportional to $\sin\varphi$. The angle φ is that formed by the line of sight with the line marking the maximium fall of temperature at different altitudes. The latter is assumed by Prof. Anderson to be the central line of the moon's shadow—*i.e.*, the line drawn to the centre of the sun—so that φ is the angular distance of a point near the sun's limb from its centre. If Prof. Anderson is right, the displacement of a star near the sun during an eclipse would, therefore, be inversely proportional to its distance from the sun's centre in accordance with the result of observation. But even with an unlimited atmosphere all would depend on a very delicate adjustment of temperature distribution. If—as would be more natural to assume—the fall of temperature were regulated not by the central line of the moon's shadow, but by the edge of the umbra, the result would be entirely different; ε would cease to be a small angle, and our equations would not be applicable.

But these considerations are beside the point, for in the denominator of (6) it is the second and not the first term that is dominating. The exponential factor of h is made up of two parts, of which h_1 depends on the diminution of density and h_2 on the diminution of the cooling effect with altitude. Neglect for a moment the latter—*i.e.*, assume the fall of temperature to be the same at all levels. In an isothermic atmosphere h would have the value of 0·125 with the kilometre as the unit of length. Taking the diameter of the penumbra to be 300 km. m is approximately 0·02, and hence mh_1^{-1} is 0·16. If the diminution of the temperature effect be taken into account, I estimate that this figure should be divided by four, and unless the sun be very near the horizon mh^{-1} will be considerably less than unity. We may therefore disregard $n^2 \sin^2\varphi$ in the denominator, as compared with h^2.

What interests us is not the actual amount of refraction, but its differences at different points near the sun, and for this purpose the best condition for a favourable effect is that made by Prof. Anderson with regard to the slope of temperature in

* Since the above was written Prof. Anderson has modified his position ("Nature," Jan. 29, 1920) and some of the criticisms applicable to his original treatment are no longer valid.

the atmosphere. We continue to assume, therefore, that θ is the zenith distance of the centre of the sun—in other words, that φ is the angular distance from the sun's centre.

The angle ψ being small, we may substitute the sine for its tangent, and calling ε' that part of ε which depends on φ we find

$$\varepsilon' = Cmnh^{-2}\tau_0 \cos mc \sin \varphi. \quad . \quad . \quad . \quad . \qquad (7)$$

When the limited height of the atmosphere is taken into account, the displacement of a star therefore increases with increasing distance from the sun, in contradiction to the observed effects. For the purpose of estimating the total amount of the effect, we may take the sun to be at an altitude of 45 deg., and the maximum fall of temperature to be 5°C. At or near the point of maximum cooling ($c=o$), (7) then becomes

$$\varepsilon' = 20Cm^2 \sin \varphi / h^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

$$= (2{\cdot}58 \times 10^{-8}) \sin \varphi\ ;$$

or, if φ is small,

$$\varepsilon' / \varphi = 2{\cdot}58 \times 10^{-8}.$$

If the distance of a star from the centre of the sun be a solar diameter—*i.e.*, 0·5 deg.—the displacement in seconds of arc is $4{\cdot}6 \times 10^{-6}$. The distance of such a star from another one equally far away on the other side of the sun would be measured by about the 230,000th part of a second.

Apart from the deflexion depending on the angular distance, there is a portion depending only on the zenith distance of the sun. This is, of course, in addition to the ordinary refraction when there is no eclipse. According to (6) this portion is greatest when sin mc is unity—*i.e.*, when the observer stands at the point where the depression of temperature alters most quickly. As ψ is a small angle, the deflexion then is for an altitude of 45 deg. : $2Cm\tau_0/h$, which amounts to about the 50th part of a second. Owing to differences in γ, the distances of two stars, one of which is above and the other below the sun, would have their apparent distances increased by an amount appreciably greater than that depending on φ.

I have also carried out the calculation for the case where the diminution of the temperature effect with altitude is represented by a linear function of h and proportional to $h-r$, vanishing at a height r. The expressions obtained are more complicated, but show that the adopted value for the diminu-

tion of the temperature effect is equivalent to the assumption that if diminishing linearly it vanishes at an altitude of 6 km.

ABSTRACT.

The Paper is an investigation of the possible deviation of the light from a star near the sun due to the temperature changes in the atmosphere produced by the passage of the moon's shadow across the earth during an eclipse. It is shown that while the actual displacements from this cause vary widely for slight differences in the assumed conditions, they are always negligibly small compared with the effects observed at the last solar eclipse.

DISCUSSION.

Mr. T. Smith said he had not grasped the necessity of assuming that the phenomena would be symmetrical about the centre of the sun. If this limitation were dispensed with in the treatment, much larger displacements might be obtained.

The Author said the symmetry was necessary to account for the actual observations.

Dr. Chree said he was not convinced that the actual observations justified the assumption of symmetry. Of the 15 or so stars on the plate, probably only five or six were of significance, and he did not think that they gave unquestionable evidence of symmetry in the displacements. Moreover, during any single eclipse, the time of day is changing, and it is impossible to get the unmixed temperature effects of the eclipse alone. A large number of eclipse observations would have to be made and compared before the evidence was regarded as conclusive. Further, he thought such observations should be made from a considerable height, as observations taken near the surface of the earth were always subject to variable local disturbances.

Mr. F. J. W. Whipple said that he did not see the justification for assuming that the pressure could be regarded as uniform. Convection could hardly equalise the pressure in the time available. It might, in fact, increase the difference. The shadow traversed the earth at a very high speed, and a pressure wave might be sent out. Temperature effects might also arise from the passage of this adiabatic disturbance through the atmosphere.

The Author agreed that the assumption of equality of pressure presented a difficulty; but he thought convection would undoubtedly reduce any differences rather than increase them.

XVI. *On Balancing Errors of Different Orders.* By T. SMITH, B.A. (*From the National Physical Laboratory.*)

RECEIVED OCTOBER 24, 1919.

THE problem to be considered may be illustrated by a somewhat artificial example. Suppose it were intended to calculate $\cos\theta$ for values of θ not exceeding one radian from an expression containing no power of θ higher than the sixth. The use of the leading terms of the expansion in infinite series

$$1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\frac{\theta^6}{6!}$$

or

$$1-0{\cdot}5\theta^2+0{\cdot}041666\ldots\theta^4-0{\cdot}0013888\ldots\theta^6$$

involves errors amounting to about 250 seventh place units. On the other hand, the use of the expression

$$0{\cdot}999{,}999{,}8-0{\cdot}499{,}993{,}7\theta^2+0{\cdot}041{,}635{,}2\theta^4-0{\cdot}001{,}339{,}9\theta^6$$

only involves errors of about 2 in the seventh place. The alterations here introduced in the numerical coefficients of the first series have the effect of compensating with fair precision for the omission of the terms in θ^8 and θ^{10}, the only other terms of importance when seven significant figures are retained. It is important for many purposes to know how the effect of such higher order terms may be most closely imitated by a series of terms of lower orders, and what magnitude the outstanding errors may reach. The series to be considered may be assumed to consist of terms involving only positive integral powers of the independent variable.

A particular case of the more general problem may be considered in the first place. Let it be required to find the unknown coefficients $a_1, a_2, a_3 \ldots a_{n-1}$ in terms of the known quantities a_n, x_1, x_2, so that

$$y=a_0+a_1x+a_2x^2+\ldots+a_{n-1}x^{n-1}+a_nx^n \quad . \quad . \quad . \quad (1)$$

lies between the closest possible limits as x varies continuously from x_1 to x_2. By substituting z for x, where z is defined by the equation

$$x-x_1=z(x_2-x_1),$$

the limits for the independent variable become 0 and 1 without any alteration in the form of y. These limits will be assumed in the following discussion.

It is not unnatural to suppose that as x changes from 0 to 1 the variation in y will be least when y is made to oscillate repeatedly between two fixed values whose difference determines the required range. The case in which all the stationary points on the curve lie between $x=0$ and $x=1$ may be considered. The limiting values for y (or one of them) are necessarily reached at the extreme points $x=0$ and $x=1$ when the range is a minimum. For, if not, y only passes through the limiting values when $x=-\varepsilon$ and $x=1+\eta$, where ε and η are positive. The limits must be reached for finite values of x, since y tends to infinity with x. Under the conditions specified a new relation can be found of the same type as (1), say $\eta=a_n\xi^n+$ a series of lower powers of ξ, where

$$\xi(1+\varepsilon+\eta)=x+\varepsilon$$

and

$$\eta(1+\varepsilon+\eta)^n=y,$$

so that η lies between smaller limits than y as ξ varies from 0 to 1 corresponding to a variation of x between $-\varepsilon$ and $1+\eta$. It follows that for minimum variation in y a limiting value is reached at both $x=0$ and $x=1$.

The two limits for y, which may be denoted by a and $-a$, since the constant term a_0 determines only the centre of the range, and does not influence its width, will be reached alternately as x increases. The values of x at which they are attained may accordingly be denoted by

$$0,\ x_2,\ x_4,\ x_6\ .\ .\ .$$

in the one case, and by

$$x_1,\ x_3,\ x_5,\ x_7\ .\ .\ .$$

in the other.

Assuming that the values $y=\pm a$ are attained the greatest possible number of times, y may be defined by the equations

$$y-a=a_nx(x-x_2)^2(x-x_4)^2(x-x_6)^2\ .\ .\ .\ (x-x_{n-2})^2(x-1) \quad (2)$$

$$y+a=a_n(x-x_1)^2(x-x_3)^2(x-x_5)^2\ .\ .\ .\ (x-x_{n-1})^2 \quad .\quad .\quad (3)$$

if n is even, or by

$$y-a=a_nx(x-x_2)^2(x-x_4)^2(x-x_6)^2\ .\ .\ .\ (x-x_{n-1})^2 \quad .\quad (4)$$

$$y+a=a_n(x-x_1)^2(x-x_3)^2(x-x_5)^2\ .\ .\ .\ (x-x_{n-2})^2(x-1) \quad (5)$$

if n is odd. The condition that the two equations of each pair determine the same function suffices to find x_1, x_2, x_3 . . . and consequently the function itself, uniquely. For in each pair of equations there are n unknowns a, x_1, x_2, . . . x_{n-1}, and

the curve given by these equations passes through $2n$ points. The conditions, therefore, just suffice to determine all the unknowns. The equations for y can easily be found from this consideration in any particular case, but for the general discussion of the problem it is preferable to derive a solution by forming the differential equation satisfied by y. Since y has in $x_1, x_2, x_3, \ldots x_{n-1}$ the full number of stationary points on the curve

$$\frac{dy}{dx}=na_n(x-x_1)(x-x_2)\ldots(x-x_{n-2})(x-x_{n-1}) \quad . \quad (6)$$

Thus, whether n be even or odd, from equations (2) to (6) it is seen that the relation

$$n^2(a^2-y^2)=x(1-x)\left(\frac{dy}{dx}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

is always satisfied. Differentiate this equation once and remove the common factor $\frac{dy}{dx}$. The resulting linear differential equation is

$$x(1-x)\frac{d^2y}{dx^2}+(\tfrac{1}{2}-x)\frac{dy}{dx}+n^2y=0. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

This is a particular case of the equation satisfied by hypergeometric series. Denoting as usual the series

$$1+\frac{a\cdot b}{1\cdot c}x+\frac{a(a+1)b(b+1)}{1\cdot 2c(c+1)}x^2+\frac{a(a+1)(a+2)b(b+1)(b+2)}{1\cdot 2\cdot 3c(c+1)(c+2)}x^3+\ldots$$

by $F(a, b, c, x)$, and remembering that the solution with a finite number of terms is alone admissible, the value found for y is

$$y=cF(n, -n, \tfrac{1}{2}, x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$=c\left\{1-\frac{n^2}{2!}4x+\frac{n^2(n^2-1^2)}{4!}(4x)^2-\frac{n^2(n^2-1^2)(n^2-2^2)}{6!}(4x)^3+\ldots\right\} \quad (10)$$

where c is a constant. Written as a descending series the value is

$$y=a_nx^nF(\tfrac{1}{2}-n, -n, 1-2n, x^{-1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$=a_nx^n\left\{1-2n(4x)^{-1}+\frac{2n(2n-3)}{2!}(4x)^{-2}\right.$$
$$-\frac{2n(2n-4)(2n-5)}{3!}(4x)^{-3}+\frac{2n(2n-5)(2n-6)(2n-7)}{4!}(4x)^{-4}$$
$$\left.-\frac{2n(2n-6)(2n-7)(2n-8)(2n-9)}{5!}(4x)^{-5}+\ldots\right\}. \quad . \quad (12)$$

From a comparison of the coefficients in (10) and (12), or from the values of (9) and (11) when $x=1$, it is seen that the coefficients c and a_n are connected by the relation

$$(-4)^n c = 2a_n. \qquad (13)$$

Equation (8) is unaltered if $1-x$ be written for x, and (9) can therefore be expressed as

$$y = c' F(n, -n, \tfrac{1}{2}, 1-x) \qquad (14)$$

The two forms (9) and (14) must be simple transformations of one another. Since the differential coefficient of

$$F(a, b, c, x) \text{ is } \frac{ab}{c} F(a+1, b+1, c+1, x),$$

it follows from the comparison of (9) and (14), and from the fact that all the coefficients are alternately positive and negative, none being zero, that all the stationary points of y are real, and lie between $x=0$ and $x=1$. The function y therefore satisfies all the conditions that have been laid down. The range of variation is $2a$, where $a=c$, the value for $x=0$. Thus, from (13) the range is that for the term in x^n alone multiplied by 4, or, apart from sign, 4^{1-n}.

It has now to be shown that y as just found lies between closer limits than any other function of the form (1) having the same coefficient for the term in x^n. The curve obtained has n branches between the limits $y=\pm a$, $x=0$ and $x=1$, the terminal points of the branches being the $n-1$ stationary points and the end points. If Y is another function varying if possible between closer limits $\pm\beta$ from $x=0$ to $x=1$, this curve must intersect every branch of the y curve. Thus, $Y-y=0$ for n values of x; but $Y-y$ is at most of degree $n-1$ in x, and thus can only vanish for n values of x if every term is zero. The limits $\pm a$ must, therefore, be exceeded at some part of the interval $x=0$ to $x=1$ by any other function of the form prescribed.

In a number of cases the problem is modified by the consideration that only odd or only even powers of the variable are admissible. The latter case is evidently solved at once by writing x^2 for x and $n/2$ for n. The discussion may proceed generally on the lines already followed, except that it may be noted that the restriction imposed means that the limits for $x=0$ to 1 will also hold for $x=0$ to $x=-1$, with or without a change of sign for corresponding distances on opposite sides of $x=0$, according as the powers are odd or even. The case

may evidently be considered by writing $x=1$ and $x=-1$ as the limits at which the values $y=\pm a$ are to be attained. The differential equation is therefore derived from

$$n^2(a^2-y^2)=(1-x^2)\left(\frac{dy}{dx}\right)^2,$$

giving

$$(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+n^2y=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

which may be re-written in the form

$$\xi(1-\xi)\frac{d^2y}{d\xi^2}+(\tfrac{1}{2}-\xi)\frac{dy}{d\xi}+\frac{n^2}{4}y=0,$$

where ξ stands for x^2. When n is even this gives

$$y=cF\left(\frac{n}{2}, \ -\frac{n}{2}, \tfrac{1}{2}, x^2\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

agreeing with the solution already suggested. This solution does not satisfy the conditions of the problem when n is odd, the number of terms being infinite. The solution to be adopted in this case is

$$y=cxF\left(\frac{1+n}{2}, \ \frac{1-n}{2}, \ \frac{3}{2}, \ x^2\right), \quad . \quad . \quad . \quad . \quad . \quad (17)$$

which has the required number of terms and is easily seen to satisfy equation (15).

It should be particularly noted that an expression like (17), though giving the minimum variation over the interval -1 to 1, does not give the least range for a series of odd powers over the interval 0 to 1. For take the simple case

$$y=3x^5-5x^3(a^2+b^2)+15xa^2b^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

which is stationary at the points

$$(a, \alpha), (b-\beta), (-a, -\alpha), (-b, \beta)$$

where $b>a$ and a, b, α are positive, and

$$\alpha=2a^3(5b^2-a^2), \ \beta=-2b^3(b^2-5a^2). \quad . \quad . \quad . \quad . \quad (19)$$

Consider in conjunction with (18) the associated function

$$\eta=3\xi^5-5\xi^3(a^2+b^2)\lambda^2+\frac{15}{4}\xi(b^2-a^2)^2\lambda^4 \quad . \quad . \quad . \quad (20)$$

which has the stationary points

$$\left\{(b-a)\frac{\lambda}{\sqrt{2}},\ (\alpha+\beta)\frac{\lambda^5}{\sqrt{2}}\right\},\quad \left\{(b+a)\frac{\lambda}{\sqrt{2}},\ -(\alpha-\beta)\frac{\lambda^5}{\sqrt{2}}\right\}$$

$$\left\{-(b-a)\frac{\lambda}{\sqrt{2}},\ -(\alpha+\beta)\frac{\lambda^5}{\sqrt{2}}\right\},\quad \left\{-(b+a)\frac{\lambda}{\sqrt{2}},\ (\alpha-\beta)\frac{\lambda^5}{\sqrt{2}}\right\}$$

The conditions for the end points are evidently

$$\alpha=3-5(a^2+b^2)+15a^2b^2 \quad \ldots\ldots \quad (21)$$

and

$$(\alpha+\beta)\frac{\lambda^5}{\sqrt{2}}=3-5(a^2+b^2)\lambda^2+\frac{15}{4}(b^2-a^2)^2\lambda^4. \quad \ldots \quad (22)$$

These equations show that if one solution of the kind considered is known another may be derived directly. The solution of the type hitherto considered is

$$a=\frac{\sqrt{5}-1}{4},\quad b=\frac{\sqrt{5}+1}{4},\quad \alpha=\beta=\frac{3}{16},$$

so that y covers the range 0·375 as x increases from 0 to 1. A comparison of the solutions of (18) and (20) shows that a self-conjugate solution is obtained when $\lambda=1$, so that

$$b=a(\sqrt{2}+1),\quad \alpha=\beta(\sqrt{2}+1),$$

the numerical values being $a=0{\cdot}33741$, $b=0{\cdot}81459$, $\alpha=0{\cdot}24615$, $\beta=0{\cdot}10196$, and the total range of variation 0·34811, which is less than in the previous solution. The principles followed in the foregoing discussion make it clear that the variation over the half range will be a minimum when one of the limits is zero, the value for $x=0$. The solution of (20) shows that this solution may be derived from the first solution of (18), since $\alpha=\beta$. The value of λ from (22) is 1·038116, giving stationary points at (0·36703, 0·31970) and (0·82070 ,0), with a range of variation 0·31970.

The expressions which have been found having the property of oscillating about zero with small excursions may be used to simplify expressions for purposes of calculation, or may be applied in a variety of ways in finding the most favourable combination of errors of different orders which may be deliberately introduced to compensate as far as possible for the presence of an irremovable error. The results already reached show that by this means the outstanding errors are greatly reduced, in many cases to an extent which renders them immeasurable. In such applications a slight variation from the foregoing cases is sometimes necessary, as the con-

ditions of the problem involve that, though both odd and even powers of the variable are admissible in general, the first power must not be present. This requirement is met by transferring the origin from $x=0$ to $x=x_1$, the first stationary point, and replacing x as independent variable by ξ, where

$$\xi=(x-x_1)/(1-x_1).$$

It is of interest in this connection to know where the stationary values occur. They are given in the following table from $n=2$ to $n=10$.

Values of xf or which y is stationary.

n									
2.	·5								
3.	·25	·75							
4.	·14645	·5	·85355						
5.	·09549	·34549	·65451	·90451					
6.	·06699	·25	·5	·75	·93301				
7.	·04952	·18826	·38874	·61126	·81174	·95048			
8.	·03806	·14645	·30866	·5	·69134	·85355	·96194		
9.	·03015	·11698	·25	·41318	·58682	·75	·88302	·96985	
10.	·02447	·09549	·20611	·34549	·5	·65451	·79389	·90451	·97553

It will be noted from the table that the stationary points are closely packed towards the ends of the range, and are widely separated near the middle. The curves thus differ very appreciably from those representing the circular functions when plotted against the angle. The calculation of the roots is simplified by noticing that if n has integral factors the equation may be resolved into a series of equations of lower orders. Thus, if $n=pq$, the qth, $2q$th, $3q$th . . . stationary points occur for the same values of x as the 1st, 2nd, 3rd . . . points in the p equation. That these results are generally true may be seen as follows :—

Let y denote $F(p, -p, \frac{1}{2}, x)$, which oscillates between $\pm a$, so that

$$p^2(a^2-y^2)=x(1-x)\left(\frac{dy}{dx}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and

$$x(1-x)\frac{d^2y}{dx^2}+(\tfrac{1}{2}-x)\frac{dy}{dx}+p^2y=0. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Further, let z denote $F(q, -q, \frac{1}{2}, y)$, transformed for the interval $y=-a$ to $y=+a$, so that by analogy with (15) z satisfies the equation

$$(a^2-y^2)\frac{d^2z}{dy^2}-y\frac{dz}{dy}+q^2z=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The variable z may be expressed as a function of x instead of y by eliminating a and y from the above three equations together with

$$\frac{dz}{dx}=\frac{dz}{dy}\cdot\frac{dy}{dx}$$

and

$$\frac{d^2z}{dx^2}=\frac{d^2z}{dy^2}\cdot\left(\frac{dy}{dx}\right)^2+\frac{dz}{dy}\cdot\frac{d^2y}{dx^2}.$$

The elimination of a between (23) and (25) yields

$$x(1-x)\,\frac{d^2z}{dy^2}\left(\frac{dy}{dx}\right)^2-p^2y\,\frac{dz}{dy}+p^2q^2z=0,$$

or

$$x(1-x)\,\frac{d^2z}{dx^2}+(\tfrac{1}{2}-x)\,\frac{dz}{dx}+p^2q^2z$$

$$=\frac{dz}{dy}\left\{x(1-x)\,\frac{d^2y}{dx^2}+(\tfrac{1}{2}-x)\frac{dy}{dx}+p^2y\right\}=0\quad.\qquad(26)$$

by (24), so that z is the polynomial corresponding to $n=pq$. The same result follows more simply by combining the two equations corresponding to (7). Now, z may be regarded as a function of y only, and is thus stationary when y is stationary. Therefore values of x for which y is stationary have the same property for z. It is easily seen that the determination of all the stationary positions for $F(n, -n, \frac{1}{2}, x)$, where n is not prime, may be resolved into the solution of a series of equations of the order of the lowest integral factor of n other than unity. In the case just considered there are q values of x on each branch of the p curve for which z is stationary. The roots may also be found simply by solving a trigonometrical equation.

The formulæ that have been obtained may be applied repeatedly to remove any number of terms from an expansion, and thus give an approximate simplified expression for a given function. As, however, the roots of $F(n, -n, \frac{1}{2}, x)$ depend upon n, the expression so obtained will not in general be the closest possible approximation to the function of that particular form; but if the series from which the approximation is derived converges rapidly the stationary positions will not differ greatly from those of the polynomial used to remove the lowest term. The knowledge of the roots of the corresponding hypergeometric series may be utilised in obtaining the closest approximation.

Assume that it is required to determine the coefficients $a_0, a_1, a_2 \dots a_n$, so that the polynomial

$$a_0+a_1x+a_2x^2+ \dots +a_nx^n$$

may represent the function $\varphi(x)$ as closely as possible throughout the interval $x=\alpha$ to $x=\beta$. It may be assumed that $\varphi(x)$ has no stationary values within this interval. The argument already applied shows that y, defined by the equation

$$y=\varphi(x)-a_0-a_1x-a_2x^2 \dots -a_nx^n$$

must be made to lie between the values $\pm\gamma$, that one or both of these values must be reached for $x=\alpha$ and $x=\beta$, and that between these limits there must be n values of x, say $x_1, x_2, x_3 \dots x_n$, at which $y^2=\gamma^2$ and $\frac{dy}{dx}=0$. Thus, writing φ for $\varphi(x)$, φ_1 for $\varphi(x_1) \dots$ and denoting differentiations by accents, the $2n+2$ conditions

$$\begin{aligned}
\varphi(\alpha)-\gamma &= a_0+a_1\alpha+a_2\alpha^2+ \dots +a_n\alpha^n \\
\varphi_1+\gamma &= a_0+a_1x_1+a_2x_1^2+ \dots +a_nx_1^n \\
\varphi_1' &= a_1+2a_2x_1+ \dots\dots +na_nx_1^{n-1} \\
\varphi_2-\gamma &= a_0+a_1x_2+a_2x_2^2+ \dots +a_nx_2^n \\
\varphi_2' &= a_1+2a_2x_2+ \dots\dots +na_nx_2^{n-1} \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\varphi(\beta)\pm\gamma &= a_0+a_1\beta+a_2\beta^2+ \dots +a_n\beta^n
\end{aligned}$$

suffice to determine $a_0, a_1 \dots a_n, x_1, x_2 \dots x_n$, and γ when φ, α and β are given. The polynomial formed by substituting these values for $a_0, a_1 \dots a_n$ is the closest possible approximation to φ of the assigned form for the interval α to β.

The method as it stands is in most cases unsuited to numerical computation. A rigorous method of treatment is possible if over the whole interval the errors resulting from the application to the usual expansion of the approximate method already described have been found. There will ordinarily be $n+2$ positions where these attain considerable dimensions, — the two end points and n intermediate stationary values. Let the magnitudes of these errors and their positions be noted.. If these errors are all of equal numerical value, but alternating in sign, the expansion obtained is the closest possible fit. Usually they vary in magnitude. It is, however, possible to write down at once a correcting expression which will make the values at these points of the required magnitude while

retaining the proper form for the approximate expression. When this is added the stationary points for the sum will be slightly displaced; but as long as the displacements are small the values at the points previously considered will not differ appreciably from those at the new stationary points, and the problem is solved to a sufficient degree of accuracy. If the errors found at $a, x_1, x_2 \ldots$ be denoted by $\varepsilon_0, \varepsilon_1, \varepsilon_2 \ldots$, the correcting factor η is readily seen to be given by the equation

$$\begin{vmatrix} -\eta & 0 & 1 & x & x^2 & \ldots & x^n \\ \varepsilon_0 & 1 & 1 & a & a^2 & \ldots & a^n \\ \varepsilon_1 & -1 & 1 & x_1 & x_1^2 & \ldots & x_1^n \\ \varepsilon_2 & 1 & 1 & x_2 & x_2^2 & \ldots & x_2^n \\ \varepsilon_3 & -1 & 1 & x_3 & x_3^2 & \ldots & x_3^n \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix} = 0$$

In most cases a simplification is possible, as the positions of $x_1, x_2 \ldots$ are very close to the stationary positions of $F(n+1, -n-1, \frac{1}{2}, x)$. If $-\eta_1$ denotes

$$\varepsilon_0 \frac{(x-x_1)(x-x_2)\ldots(x-x_n)(x-\beta)}{(a-x_1)(a-x_2)\ldots(a-x_n)(a-\beta)}$$
$$+\varepsilon_1 \frac{(x-a)(x-x_2)\ldots(x-x_n)(x-\beta)}{(x_1-a)(x_1-x_2)\ldots(x_1-x_n)(x_1-\beta)},$$
$$+\ldots$$

it is evident that η_1 assumes the values $-\varepsilon_0, -\varepsilon_1, -\varepsilon_2 \ldots$ at $a, x_1, x_2 \ldots$ and would thus remove the existing errors at these points. This expression cannot, however, be used as a correcting factor, since it is of the order $n+1$ in x. If $x_1, x_2, x_3 \ldots x_n$ may be assumed to be the stationary points of $F(n+1, -n-1, \frac{1}{2}, x)$, η may be at once reduced to the next order without violating the other conditions by adding a suitable multiple of this function. The correcting term is thus

$$-\Sigma\varepsilon_\lambda \frac{(x-a)(x-x_1)\ldots(x-x_{\lambda-1})(x-x_{\lambda+1})\ldots(x-x_n)(x-\beta)}{(x_\lambda-a)(x_\lambda-x_1)\ldots(x_\lambda-x_{\lambda-1})(x_\lambda-x_{\lambda+1})\ldots(x_\lambda-x_n)(x_\lambda-\beta)}$$
$$-\frac{1}{2(-4)^{-n}}(F)\,\Sigma \frac{\varepsilon_\lambda}{(x_\lambda-a)\ldots(x_\lambda-x_{\lambda-1})(x_\lambda-x_{\lambda+1})\ldots(x_\lambda-\beta)}$$

if (F) denotes $F\left(n+1, -n-1, \frac{1}{2}, \dfrac{x-a}{\beta-a}\right)$

where a and β are usually 0 and 1 respectively.

From the high order of accuracy attained in typical examples, it is a simple deduction that the mathematical expression of a law discovered from experimental results, in spite of a high order of agreement, cannot safely be taken as a true representation of the facts unless the agreement extends over a very great interval.

It is evident that the expansion of an arbitrary function $\varphi(x)$ in the form

$$\varphi(x)=a_0F_0+a_1F_1+a_2F_2+\ \ldots$$

where F_n denotes $F(n,\ -n,\ \frac{1}{2},\ x)$ may prove of considerable importance. To derive such an expansion the identity

$$\tfrac{1}{2}(4x)^n=\frac{(2n)!}{(n!)^2}F_0-\frac{(2n)!}{(n-1)!(n+1)}F_1$$
$$+(-)^p\frac{(2n)!}{(n-p)!(n+p)!}F_p+\ldots+(-)^nF_n. \quad (27)$$

may be employed. The special convention $F_0=\frac{1}{2}$ has been introduced. If then $\varphi(x)$ is required from $x=h$ to $x=h+1$, Taylor's theorem gives

$$\varphi(h+x)=\varphi+x\varphi'+\frac{x^2}{2!}\varphi''+\frac{x^3}{3!}\varphi'''+\ \ldots$$

where the general functions on the right depend upon h only. The substitution for x^n from (27) gives

$$\begin{aligned}\tfrac{1}{2}\varphi(h+x)=F_0&\left\{\varphi+\frac{2!}{(1!)^3}\frac{\varphi'}{4}+\frac{4!}{(2!)^3}\frac{\varphi''}{4^2}+\frac{6!}{(3!)^3}\frac{\varphi'''}{4^3}+\ \ldots\right\}\\ -F_1&\left\{\frac{2!}{0!1!2!}\frac{\varphi'}{4}+\frac{4!}{1!2!3!}\frac{\varphi''}{4^2}+\frac{6!}{2!3!4!}\frac{\varphi'''}{4^3}+\ \ldots\right\}\\ +F_2&\left\{\frac{4!}{0!2!4!}\frac{\varphi''}{4^2}+\frac{6!}{1!3!5!}\frac{\varphi'''}{4^3}+\ \ldots\right\}\\ -F_3&\left\{\frac{6!}{0!3!6!}\frac{\varphi'''}{4^3}+\frac{8!}{1!4!7!}\frac{\varphi''''}{4^4}+\ \ldots\right\}. \quad (28)\\ &+\ \ldots\end{aligned}$$

The coefficient of every F is finite if the Taylor series is convergent. The arrangement of the coefficients in this form may evidently be discontinued at any stage, and the Taylor expansion substituted. The corresponding formula for use between the limits $x=\pm 1$ may be written down at once. For the hypergeometric function of equation (11) terminates when n is the half of any positive integer, and therefore if $\frac{n}{2}$ is written for n

this formula includes both (16) and (17). The alterations required in (28) are now obvious.

The analogy between (28) and Fourier's series should be noted. If x is replaced by suitable circular functions the F's represent other circular functions. Thus

$$F\left(n,\ -n,\ \tfrac{1}{2},\ \frac{1-\cos\theta}{2}\right)=(-)^n F\left(n,\ -n,\ \tfrac{1}{2},\ \frac{1+\cos\theta}{2}\right)=\cos n\theta.$$

If n is even

$$F\left(\frac{n}{2},\ -\frac{n}{2},\ \tfrac{1}{2},\ \sin^2\theta\right)=(-)^{\frac{n}{2}} F\left(\frac{n}{2},\ -\frac{n}{2},\ \tfrac{1}{2},\ \cos^2\theta\right)=\cos n\theta,$$

and if n is odd

$$n\sin\theta\, F\left(\frac{1+n}{2},\ \frac{1-n}{2},\ \frac{3}{2},\ \sin^2\theta\right)=\sin n\theta.$$

These relations indicate that the series coefficients in (28) may be replaced by integrals when summed to infinity. They may be so expressed in many different forms. The parallel shows that as with Taylor's series so with Fourier's series, a closer approximation when a finite number of terms only is retained may be obtained by modifying the coefficients from the values given by integration. The closest possible approximation is obtained when the end errors and all the maximum and minimum errors are numerically equal. The coefficients may be corrected in a similar manner to that employed for Taylor's series. The correcting term in this case may be written

$$\frac{1}{n+1}\Sigma(-)^\lambda \varepsilon_\lambda (F_n f_0+F_{n-1}f_1+F_{n-2}f_2+\ \ldots\ +F_0 f_n),$$

where f is written for F when x is replaced by x_λ, or

$$\frac{1}{n+1}\Sigma\varepsilon_\lambda (F_0 f_0+F_1 f_1+F_2 f_2+\ \ldots\ +F_n f_n),$$

the halving factor applying to extreme terms. The parallel with the ordinary Fourier procedure is evident.

When the function can be calculated simply by a direct method, and it is only intended to fit a given kind of formula to it, the ε's may be the actual values of the function at the stationary points. The same result is obtained by retaining a suitable number of terms in equation 28. Only those terms of the numerical coefficients corresponding to the terms neglected in the Taylor expansion need be calculated, the corrections alone being thus evaluated.

ABSTRACT.

In calculating functions from Taylor expansions or otherwise, the results obtained by summing any finite number of terms will differ to a greater or less extent from the true results. It is shown in the Paper that by suitable modifications of the coefficients the results obtained, even when comparatively few terms of the expansion are taken, can be made to approximate very closely to the true results for all values of the variable between selected limits.

DISCUSSION.

Mr. F. J. W. WHIPPLE asked why, when terms of higher order than n were to be neglected, the author found it advantageous to use $n+2$ points rather than $n+1$.

Mr. SMITH said it could be shown that the maximum accuracy was obtainable by taking $n+2$ points and giving equal and opposite errors at alternate points. He added that it was possible by the method of the Paper to represent a function which it was impossible to expand. As an example he mentioned the common logarithms from 1 to 10. Any ordinary expansion would be divergent over this range.

XVII. *Notes on a Method of Testing Bars of Magnet Steel*

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Introduction.

The present Paper is an outline of a series of tests on magnet steels carried out several years ago. Ewing's double permeameter method* for round bars was adopted, a modification being introduced in the fitting of the yokes to the bars. Although this method gives results which are sufficiently accurate for most practical requirements, it is more laborious and less accurate than that developed by Messrs. Campbell and Dye,† in which differential search coils are used to measure the value of the magnetising force. The latter method possesses the advantage that the value of the flux density and the corresponding value of the magnetising force can be measured at any part of the bar. In Ewing's method the value of B is not uniform along the bar, owing to leakage between the bars in the two limbs of the permeameter. Since leakage occurs with both the long and the short permeameters, there is a certain amount of compensation, when the B-H curve is obtained by the method outlined by Ewing, as will be shown later when dealing with the correction to be applied for leakage. Moreover, the value of B being found experimentally with a search coil at the centre of the bar (where the leakage is small), and that of H (at the centre), calculated from the permeameter constants, it follows that, provided the leakage effect was the same for both permeameters, the values of B and H would be correct. The leakage effect is not the same for both permeameters, and the value of H as found by calculation is in error. The error for any of the bars tested, which were of low permeability, does not exceed 1 per cent. As a method of precision the above has little to recommend it, whilst the additional labour required in taking two sets of readings, combined with the fact that the final result is only found after reduction from two B-H curves, is such as to make it inferior to the differential coil method.

* "Magnetic Induction in Iron and other Metals," "The Electrician," Vol. XXXVIII., p. 110, 1896.

† "Journal," I.E.E., Vol. LIV., p. 35, 1915.

Description of Apparatus.

A plan and elevation of the long permeameter is shown in Fig. 1. The short permeameter is the same in every respect as that in Fig. 1, excepting that the length of the winding is 3½ in. instead of 7. Six layers of 324 series turns are wound on each limb, the maximum value of H obtainable being 450 C.G.S. with a current of 20 amperes. The coils are wound

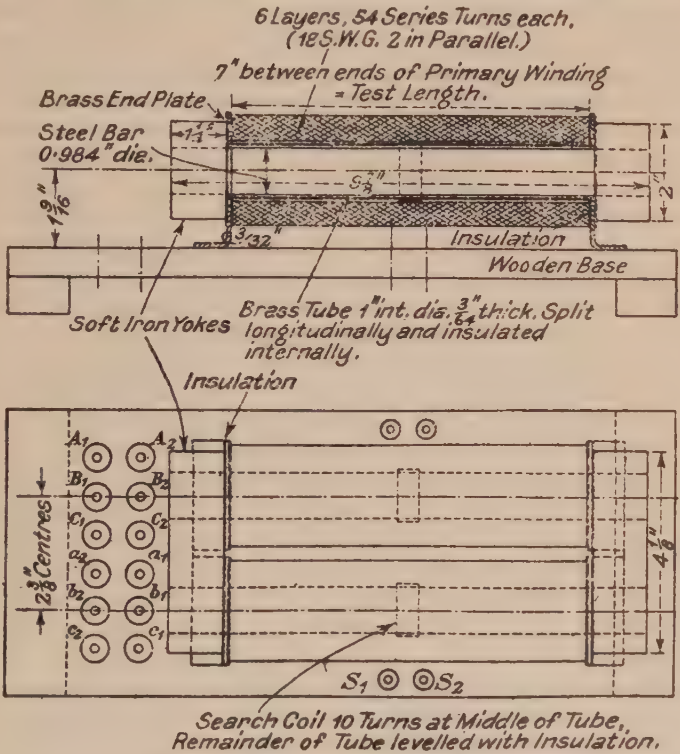

FIG. 1.—PLAN AND ELEVATION OF LONG PERMEAMETER P_1.
A_1A_2=Terminals of first layer. 54 turns ($H=3{\cdot}82I$).
B_1B_2=Terminals of second layer. 108 turns.
C_1C_2=Terminals of third layer. 162 turns (H total$=22{\cdot}92I$), where I=current.

on brass tubes fitted with end plates, split longitudinally to minimise eddy current effects. The space in a tube due to the removal of metal was filled with insulation to prevent the ends being drawn together when the coils were being wound. Although no deleterious effects were observed as a result of

the use of brass, it is better to use an insulating material on which to wind the coils. Such a material is not likely to be so permanent, of course. The terminals are arranged so that one or more layers can be used, and in this way small values of H are obtained without using a very small current.

The bars tested were nominally 1 in. diameter as taken from the rolls. These were ground down by means of a precision grinding machine, to 0·984 in. diameter, thus removing the material which had been in immediate contact with the rolls. The cuts taken by the grinder were less than 1/1,000 in., and the mass of metal, beneath the surface, affected by the grinding was the least possible. Two yokes of very soft and permeable iron were accurately bored to fit the bars, so that the former could easily be drawn on (the bars being slightly lubricated) by means of a small screw jack. In this way the air gap between the bars and yokes was reduced to a minimum, and the necessity for pinching screws, whereby a variable amount of stress is applied to the bars when tested with different permeameters, was therefore eliminated. In making tests with two permeameters, it is essential that the reluctance of the yokes and joints should be the same for each permeameter for any given value of the flux density. Accurate machining of the bars and yokes is necessary to secure this condition. Tests were made with both permeameters by putting a mark on each bar and rotating the bar to a different position for the various tests. The results obtained in this way were in agreement within the limits of experimental error. When the bars are magnetised there is a force between them and the yokes, which tends to reduce the air gap. This introduces a certain amount of compressive stress, which will affect the magnetic properties of the materials where it occurs. For any given value of B, however, this condition is the same for both permeameters and the effect does not make its appearance in the corrected B-H curve.

The apparatus required to obtain B-H curves and hysteresis loops is shown diagrammatically in Fig. 2. The ammeter A could be used to read from 1/1,000th to 20 amperes, using four different shunts all of which were enclosed within the instrument case. The B-H curves were obtained by the method of reversals using a moving-coil ballistic galvanometer having a free period of 12 seconds. The calibration of the galvanometer was effected by means of a standard solenoid about 2 metres long, having a search coil of 400 turns, situated at its centre,

and giving up to 5×10^6 interlinkages, *i.e.*, line turns, on reversal of the current. The search coil of the solenoid was permanently connected in the galvanometer circuit, so that the conditions under which the *B-H* curves were taken were identical with those when the galvanometer was calibrated, except that the iron was removed from the permeameter for the latter operation. In order to destroy any residual magnetism in the yokes, each yoke was subjected to a few sharp blows before being fitted to the bars.

It was thought that errors might be introduced owing to time lag of the flux in the interior of bars nearly 1 in. diameter,

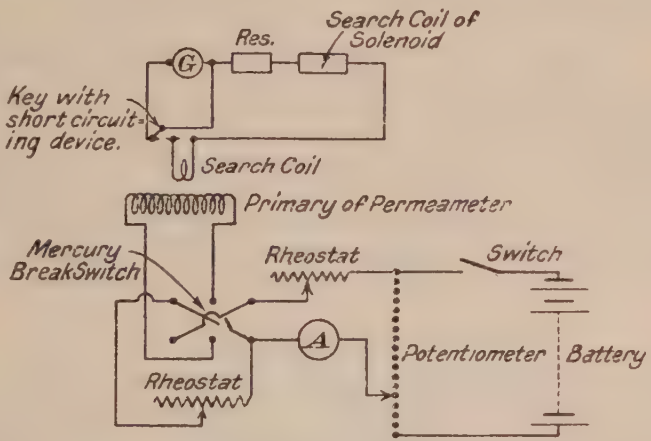

FIG. 2.—DIAGRAMMATIC SKETCH OF APPARATUS FOR OBTAINING *B-H* CURVES AND HYSTERESIS LOOPS.

thereby giving a smaller galvanometer throw than if the flux change had been transient. This point was investigated by using a Grassot fluxmeter to measure the flux change on reversal of various values of the current and comparing the results with those obtained with the galvanometer under identical conditions. The agreement between the results found by the two methods was accurate within the limits of errors of observation, although the flux change, especially on the steep portion of the *B-H* curve, with bars of relatively large permeability, was not complete before the galvanometer coil began to move (*see* Appendix).

Meaning of Symbols Used.

B = flux density in lines per square centimetre.

B_{rem} = Remanence, *i.e.*, the value of B on the hysteresis loop for which $H=O$.

J = Intensity of magnetisation $=(B-H)/4\pi$.

$J_{rem.}$ = Remanent intensity of magnetisation on the hysteresis loop for which $H=O$.

$=B_{rem.}/4\pi$.

H = Magnetising force in C.G.S. units.

H_c = Coercive force, *i.e.*, the negative value of H required to reduce the remanent intensity of magnetisation $J_{rem.}$ to zero, when proceeding from $J_{max.}$ round a hysteresis loop. In the case of magnet steels the slope $\left(\frac{dB}{dH}\right)$ of the hysteresis loop in the neighbourhood of $B=O$, is sufficiently steep that the negative value of H corresponding to $B=O$, is almost equal to that when $J=O$. The difference between the two values of H falls within the limits of experimental error. In these tests, therefore, the coercive force has been taken as the negative value of H corresponding to $B=O$. The above definitions are almost identical with those used by Messrs. Campbell and Dye.

Treatment of Bars Before Testing.

In carrying out tests on magnet steels using the double permeameter method, it is essential that the treatment of the bars should be the same before being tested by individual permeameters, *i.e.*, the bars should have the same magnetic history. Each bar was, therefore, placed in a solenoid about 15 in. long, through the winding of which an alternating current of from 1 to 2 periods per second was passed for about 1 minute and then gradually reduced to zero. The condition of the bars was tested by slipping off a coil and observing the throw on a ballistic galvanometer connected to it. By this means it was possible to reduce the remanence ($B_{rem.}$) below 30 lines per square centimetre. To secure reliable results, the magnetic qualities of each of the pair of bars should be in close agreement, and in the experiments herein described, the permeability of individual bars of each pair differed by less than 1·5 per cent. By connecting the search coils on the two limbs of the permeameter in series, an average of the two bars is obtained.

Leakage.

Both permeameters were tested for magnetic leakage by winding a search coil, having the same number of turns as that on one of the brass tubes, on each yoke. The leakage value of B was ascertained by connecting the search coil on a yoke in opposition to that on one of the brass tubes, raising the sensitivity of the galvanometer by cutting resistance out of the secondary circuit and noting the throw on reversal of the current. Assuming that the rate of change of flux $\left(\frac{dB}{dt}\right)$ is the same through each coil, the throw is a measure of the flux which does not pass through the centre of the yoke, *i.e.*, the leakage flux.* The leakage can also be found by observing separate galvanometer throws with each coil.

Let δ_1=deflection with search coil on bar.

Let δ_2=difference in deflection with search coil on bar and that on yoke; then leakage fraction$=\frac{\delta_2}{\delta_1}=\lambda$.

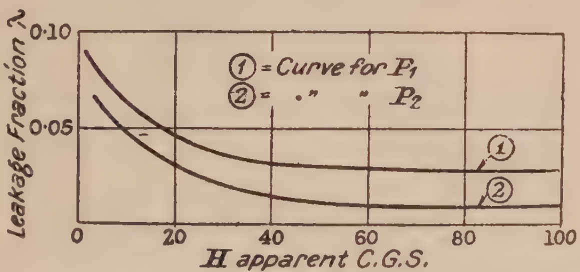

FIG. 3.—DIAGRAM SHOWING LEAKAGE FRACTION PLOTTED AGAINST APPARENT VALUES OF H, FOR BOTH PERMEAMETERS.

The circuits followed by leakage lines and lines through the yoke are in parallel; thus the same value of H produces the lines in both circuits. Since P_1 (long permeameter) is twice as long as P_2 (short permeameter), the leakage will be greater with the former than with the latter. This is borne out by experimental results, as illustrated in Fig. 3.

* The value of B can be made almost uniform throughout the whole magnetic circuit by means of compensating coils at the ends of the permeameter limbs. The magnetising force due to these coils, however, affects the value of H at the centre of the bars by an amount which cannot be accurately calculated. The use of compensating coils on both permeameters would modify the two sets of results making it impossible for the true values of H to be easily and accurately found, since the test length with P_1 is twice that with P_2.

If the correction applied to find the "true" value of H at the middle of the bar is comparatively large, the effect of leakage may be appreciable. As the leakage is different for each permeameter, the reluctance of the paths (including leakage paths) other than the test lengths will also be different. Thus the additional ampere turns necessary to overcome this reluctance will not be the same for each permeameter, and the corrected value of H obtained from the curves, as shown in Fig. 7, will not be the "true" value of H. The error from this cause is most liable to occur on the steep portion of the B-H curve, where the correction for the effect of the yokes and air-gaps is large compared with the true value of H. It will

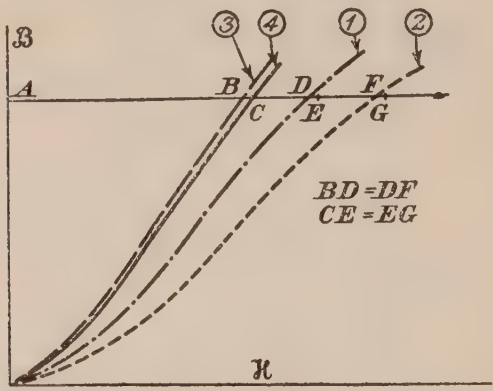

FIG. 4.

(1) Curve obtained with long permeameter.
(2) Curve obtained with short permeameter.
(3) Curve obtained from (1) and (2) by setting back DF from D.
(4) Curve obtained with zero leakage, *i.e.*, the true B-H curve. E and G are points in which curves (1) and (2) would cut AN for zero leakage.
BC=correction to be applied to (3) to obtain (4).

be seen from Fig. 3 that the leakage is greatest as low values of H, and therefore at low flux densities. This is due to the fact that the yokes are being worked at very low permeability. It would not be advantageous to decrease the cross-sectional area of the yokes, thereby working them at higher permeability, since this would necessitate large corrections for high values of B, an increase in the leakage and a diminution in the largest value of the corrected magnetising force obtained with the permeameters.

Owing to the complex nature of the leakage, it is difficult to estimate its effect on the "true" value of H to any degree of precision. In the following an attempt has been made to calculate approximately the correction to be applied in order to obtain the true value of H.

Correction for Leakage.

If it be assumed that B is proportional to H for the portions of the circuit other than the test lengths, a leakage fraction entails a corresponding diminution in the magnetising force necessary to overcome the reluctance of these portions, due to the decrease in B through them. If the true value of this force, *i.e.*, for zero leakage* is H, the actual value required with the permeameter is $H(1-\lambda)$. Thus the distance DF between curves (1) and (2) is in error by an amount depending, among other things, on λ_1 and λ_2, these being the leakage fractions with P_1 and P_2 respectively.

An approximate correction to be applied to curve (3), [obtained from (1) and (2)] in order to find the true curve (4), can be derived as follows :—

$$CD=CE(1-\lambda_1)$$

$$CF=2CE(1-\lambda_2) \qquad CF=2CD\,\frac{(1-\lambda_2)}{1-\lambda_1}$$

$BC=DF-CD$, since $BD=DF$;

$$=CF-2CD$$

$$=2CD\left(\frac{\lambda_1-\lambda_2}{1-\lambda_1}\right)$$

$$=2BD\left(\frac{\lambda_1-\lambda_2}{1+\lambda_1-2\lambda_2}\right) \text{ substituting } BD-BC \text{ for } CD;$$

$=2BD(\lambda_1-\lambda_2)$ approx., since $(\lambda_1-2\lambda_2)$ is small, compared with unity.

Now $AC\doteqdot AB$, and hence the percentage error on the true value of $H\doteqdot\dfrac{100(\lambda_1-\lambda_2)}{a}$, where $AB=2aBD$, and $\lambda_1\lambda_2$ are the leakage fractions for the apparent values of H (corresponding to the value of B under consideration) found with P_1 and P_2. It is of interest to observe that when $\lambda_1=\lambda_2$ the error vanishes. Using the above formula and the curves shown in Fig. 3, the error in the corrected value of H (as found by the method

* This, of course, can never be realised in practice.

shown in Fig. 7) for the bars used, does not exceed 1 per cent. at any point on the *B-H* curve. As it is impossible to produce magnet steel in which the magnetic qualities *always* agree within 1 per cent., this error can be disregarded.

B-H Curves, Hysteresis Loops, Remanence and Coercive Force.

After carefully demagnetising a pair of bars, as explained above, the *B-H* curve was taken with P_1. The bars were then demagnetised in a similar manner and the *B-H* curve taken with P_2. Having obtained these *B-H* curves, the corrected *B-H* curve was found by setting back the horizontal distance between the curves, so that *AB* is equal to *BC*, as shown in Fig. 7. This curve requires further correction, since the area of the search coil on the brass tube is 1·4 times the area of the bar. It is necessary, therefore, to substract 0·4 *H* from the

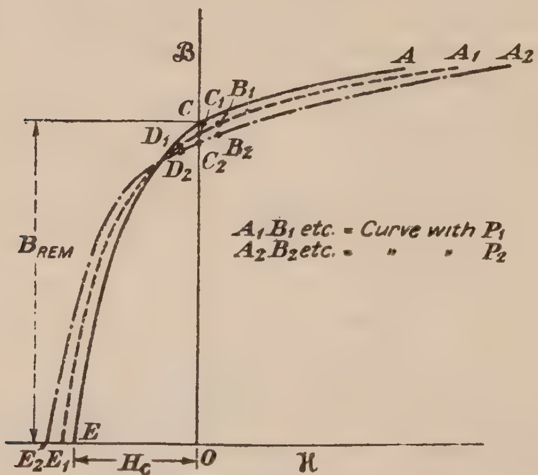

Fig. 5.—Diagram Illustrating the Method of Finding H_c^1 and B_{rem}.

ordinates of the curve, *H* being the value read on the curve found by the above procedure. Messrs. Campbell and Dye* substract this amount, automatically using a mutual inductance, the primary of which carries the current through the permeameter winding (or a fraction of it, using a shunt), the secondary being in series with the search coil. This procedure was

* *Loc. cit.*

inapplicable in the present case because the current through the primary of the permeameter is proportional to the "apparent" H and not to the corrected H.

In order to get the corrected value of $H_{max.} = 100, 200$ and 400 for the hysteresis loops, the "apparent" values of H required for P_1 and P_2 were taken from the B-H curves. Complete hysteresis loops were not taken for $H_{max.} = 200$ and 400, but five points, viz., A, B, C, D and E, were determined with each permeameter (Fig. 5). A corresponds to the maximum value of H, B and D to points near the B axis, *i.e.*, near the point $B_{rem.}$, while C corresponds to the "apparent" remanence. The corrected value of $B_{rem.}$ can then be found

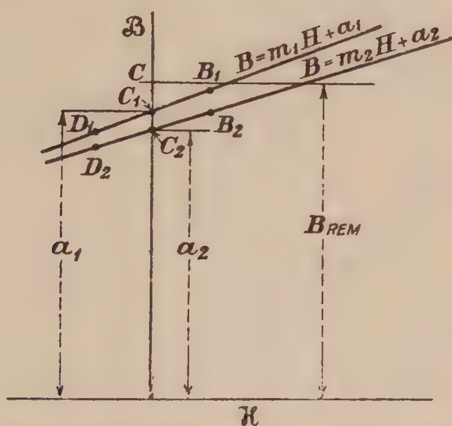

FIG. 6.—DIAGRAM ILLUSTRATING THE METHOD OF CALCULATING $B_{rem.}$.

quite accurately as shown in Fig. 5, by plotting the points B, C, D for each permeameter, and setting back the difference between the curves. The point in which the corrected curve cuts the B axis is the corrected value of $B_{rem.}$. Since the points are almost collinear $B_{rem.}$ may be found by calculation, using the co-ordinates of the points B and D. The equations to the lines through the respective pairs of points are given in Fig. 6. If m_1 and m_2 are the slopes of these lines, we have

$$B_{rem.} = \frac{2m_1 A_2 - m_2 A_1}{2m_1 - m_2}$$

H_c' is found by subtracting E_1E_2 from OE_1 (*see* Fig. 5).

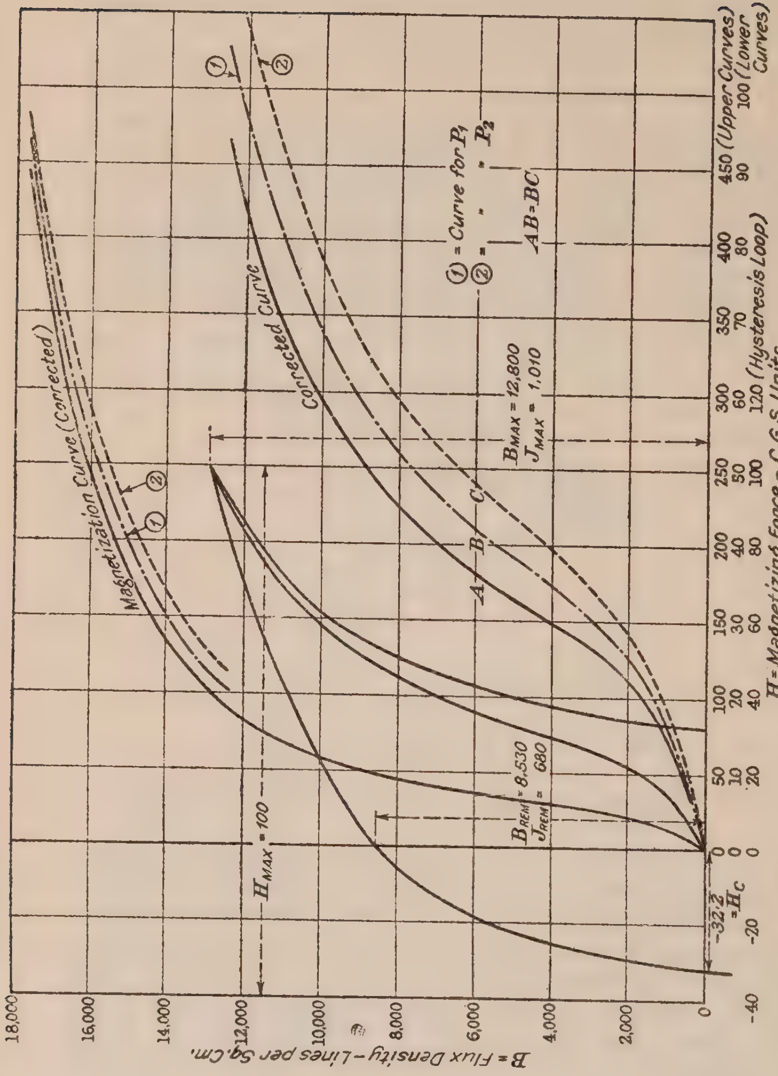

FIG. 7.—DIAGRAM SHOWING B-H CURVE AND HYSTERESIS LOOP OBTAINED WITH TWO SIMILAR BARS OF MAGNET STEEL.

In conclusion, the author wishes to convey his best thanks to Prof. E. W. Marchant, D.Sc., and to the Council of the Liverpool University for the facilities provided to enable the work to be carried out.

ABSTRACT.

In the Paper tests on cylindrical bars of magnet steel 1 in. diameter and 10 in. long are described. The tests were conducted using a slight modification of Ewing's double permeameter method. Instead of employing pinching screws in the yokes, the bars and yokes were ground accurately to 1/1,000th in., and arranged to make a good push fit. The method is compared with that in which use is made of differential coils for measuring the value of the magnetising force *in situ*, as developed at the National Physical Laboratory. It is shown that tests with the latter method can be conducted more speedily and accurately than with the double permeameter method. The variation in the magnetising force along the bar and the leakage between the pairs of bars in the permeameter is treated. A formula is developed, by means of which, with the aid of experimental data given in the Paper, a correction can be applied to allow for leakage effects. A *B-H* curve and hysteresis loop are given for a certain sample of magnet steel ; also the details of the permeameter.

DISCUSSION.

Dr. D. Owen said it was evident that the leakage did not occur only at the yokes, but began near the centre of the bars. The applied field would therefore be strongest at the centre, so that taking the average field, as done by Ewing, would give too low a value.

XVIII. *On the Forces Acting on Heated Metal Foil Surfaces in Rarefied Gases.* By GILBERT D. WEST, *M.Sc.* (*Lond.*).

RECEIVED DECEMBER 18, 1919.

Arrangement of Paper.

Introduction.

THE present research arises out of two previous Papers to the Physical Society.* Both were attempts to measure the pressure of light by a very simple method. A strip of gold leaf was suspended vertically in a test-tube, and, on exposure to radiation, the strip was deflected through a small angle. A measurement of this deflection was made by a microscope, and when the weight of the strip was known, the radiation pressure could be calculated.

It was thought that the customary "radiometer effects" by which most light-pressure measurement are disturbed, would here be very small, owing to the almost negligible difference in temperature of the two surfaces of the strip. Experiment showed, however, that although the ordinary radiometer effects were apparently absent, new gas action effects appeared. It was established that symmetrical placing of the strip had much to do with their elimination, and, at certain gas pressures, reasonably accurate measurements of the pressure of light were possible. Nevertheless, in spite of this, a separate investigation of such effects was felt to be necessary. Chiefly with a view, therefore, to the elimination of troublesome sources of error, the present work was started.

It has since been found, however, that the nature of the gas action in itself is interesting, and that, as the whole problem of the equilibrium conditions in a rarefied gas is involved, offers a wide scope for research. A considerable amount of such research has already been done. Thus, in addition to the classic experiments of Crookes, there is much careful work by the Danish physicist Knudsen.† Moreover, a number of physical instruments, such, for instance, as the pressure gauges

* "Proc." Phys. Soc., XXV., p. 324, 1913, and XXVIII., p. 259, 1916.
† "Ann. d. Phys.," 1910 and 1911.

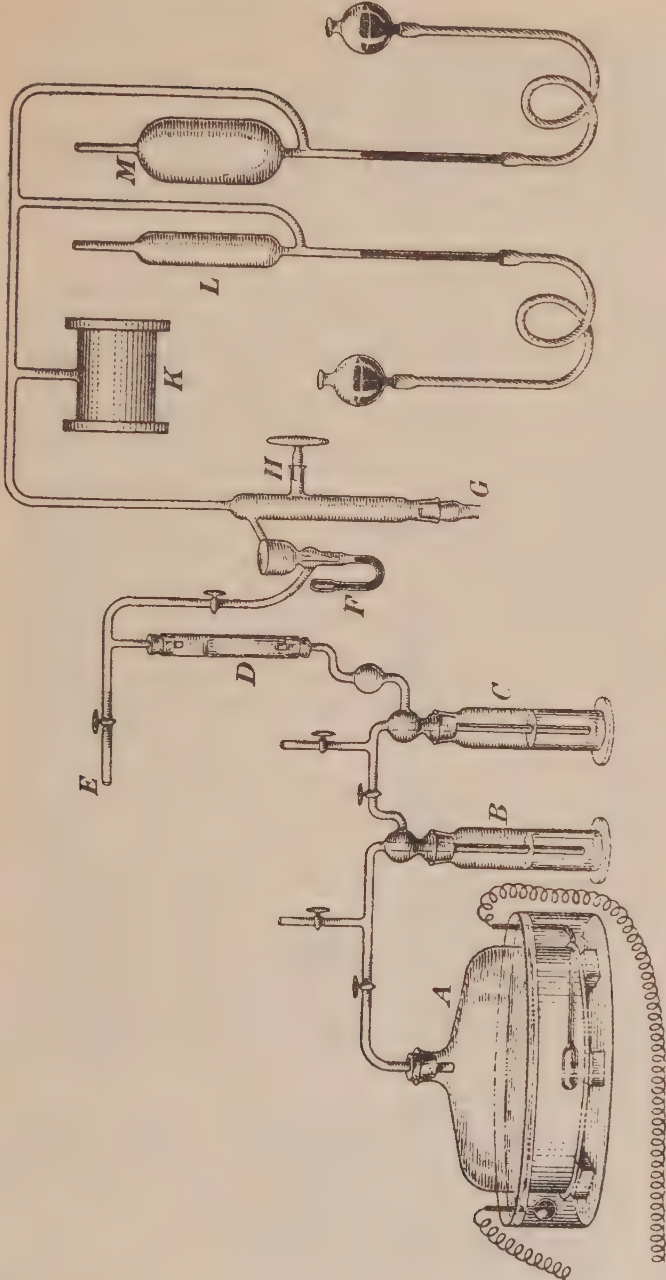

Fig. 1.

of Shrader and Sherwood,* and the thermo-galvanometers of Williams and Eccles† depend on gas action effects. Further, there are experimental results, such as those of Bottomley and King,‡ which still await explanation.

A number of wide issues thus arise, but the present Paper, at any rate, will be restricted to an experimental inquiry into the nature of the forces acting on heated strips of foil in rarefied gases.

Apparatus.

The apparatus here employed does not differ very materially from that previously used. A sketch is shown in Fig. 1. *K* represents the box in which the experiments were conducted. The gas pressure within it can be measured by the McLeod gauges *L* or *M*, *M* being used for the lowest pressures. Two pumps were used to reduce the pressure. *E* was connected to a rotary "backing" pump, whilst *G* was connected to a Gæde mercury pump. *H* is a phosphorus pentoxide bulb, and *F* is a mercury trap, designed to cut out the backing pump when a pressure of about 1 cm. of mercury is reached. Air could be admitted by turning the tap on the vertical tube between *B* and *C*, but before it reached the box *K* it was made to pass through strong sulphuric acid in *C*, and then over the phosphorus pentoxide in the tube *D*. Hydrogen could be generated electrolytically at *A*, but before being admitted to the apparatus, was, in addition, forced to pass through alkaline pyrogallol in *B*.

In the early experiments the box *K* was rectangular in shape, and measured 24 cm. long by 8 cm. broad by 8 cm. high. Owing chiefly to the fact that it was made of four separate pieces of brass, however, minute leaks occurred, and it was found difficult to get reliable observations below a gas pressure of about 0·0005 cm. mercury. A large number of observations were taken, but nearly all the experiments here recorded were made in a box constructed of a cylinder of brass, to whose ends two glass plates, cut from the same sheet, had been fixed by means of a mixture of wax and resin. The box was 9 cm. in diameter and 6 cm. long.

The strips of foil were mounted in ways shown in Figs. 2, 3, 4 and 5, and described separately in the accounts of the various experiments. They were always illuminated, however,

* "Phys. Rev.," XII., 1918, p. 70.
† "Proc." Phys. Soc., Vol. XXX., p. 253, 1918.
‡ "Proc." Roy. Soc., A. 79, p. 285, 1907.

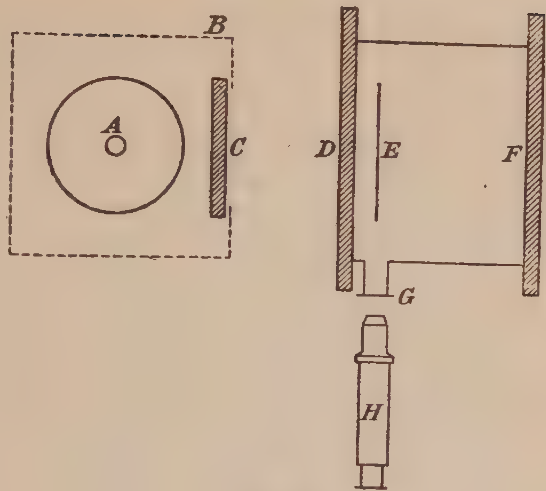

Fig. 2.

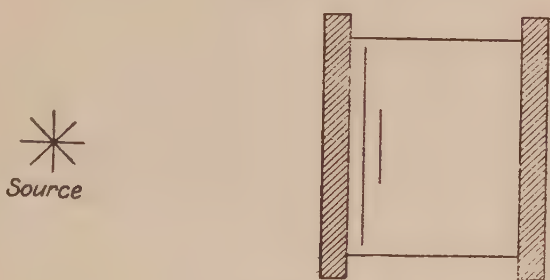

Fig. 3.

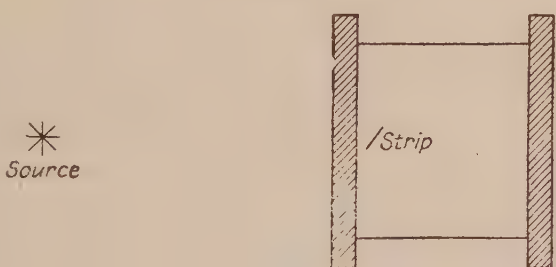

Fig. 4.

Fig. 5.

by a 500 watt " Atmos " lamp A enclosed in a metal box B, represented in Fig. 2. The walls of the box were thickly coated with lamp black, and the window C was made of glass cut from the same sheet as that used for the ends D and F of the cylindrical box. A calculation based on the shape of the metal filament, showed that it was justifiable to apply the inverse square law even as close as the glass of the bulb.

The deflection of the end of the strip was observed by means of a microscope magnifying about 30 times. In the later experiments observations were made through a small window G in the side of the vessel, but in the earlier experiments a periscopic arrangement of prisms served instead. The mean distance of the strip from the glass was measured by means of a microscope working in front of the window D. The microscope was first focussed on the inner surface of D, and then on the strip, and the forward movement of the objective measured. As the strips were never quite straight, a mean of several readings was taken. With the strip in front of the screen, as in Experiment VI., no such accurate measurement was possible.

Experimental Results.

The present order of description of the experiments has been adopted, chiefly because it lends itself easily to subsequent theoretical treatment. The order is in no sense historic.

It should further be remarked that the deflections of the strip produced by the pressure of radiation only introduce themselves in this work as very small corrections, and in general can be entirely neglected.

Experiment I.—Strips of foil, of various widths, were suspended by horizontal glass fibres attached to glass distance pieces fixed to the front window. The arrangement was similar to Fig. 2. If the box was filled with air at atmospheric pressure, it was found that the incidence of radiation caused a slight movement of the strip away from the nearest glass wall, followed by a stronger movement towards it. Small changes in inclination to the vertical and in the shape of the strip seemed to make very appreciable differences to the magnitude of the second movement. Moreover, a very considerable time elapsed before the maximum deflection of the strip was reached. As the gas pressure was lowered the movement became less and less, but at the same time it established itself rather more quickly, and thus to an extent masked the first movement. Finally, at a pressure of about 1 cm. of mercury there was

hardly any movement at all. With hydrogen gas, even at atmospheric pressure, only very slight movements were observed.

For reasons given in the section on the theoretical discussion of results, it is supposed that the second movement referred to above was caused by the establishment of convection currents. The first movement is most probably connected with the type of movement that predominates at pressures below 1 cm. of mercury. Further experiments refer to such pressures.

Experiment II.—The extreme thinness of the foils used would in itself suggest that the movements of the strips were

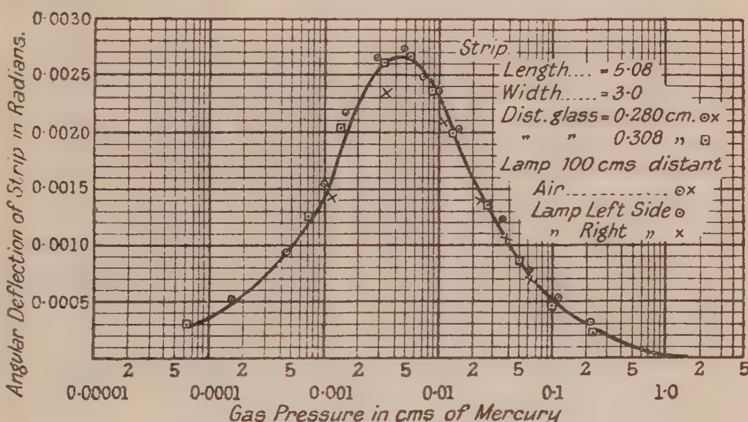

Fig. 6.—Deflections of Copper Strip No. III. in Cylindrical Brass Box. Graphical Record of Results.

independent of the difference of temperature of their surfaces. To make sure, however, the lamp was placed first one side of the box and then on the other. Examples of deflections obtained in this way are shown in Figs. 6 and 7.* It will be seen that with increasing rarefaction, the deflection of the strip rises, reaches a maximum, and finally falls. Nevertheless throughout it will be noticed that on whichever side the lamp is placed, the deflections are approximately the same. Many other such readings were taken, but on the average, still no certain difference in the two sets could be detected.

* Semi-logarithmic paper has been used owing to the great range of the gas pressures.

Experiment III.—The previous experiment shows that the extremely small difference of temperature of the two surfaces at the strip is not instrumental in producing the deflections observed. Hence the forces acting on the strip must arise in a different way from those that produce a movement of the vanes in a Crookes radiometer, and it is natural to inquire if the pressure distribution is also different. On a Crookes vane, it is supposed that the excess pressure is restricted to a narrow margin near the edge, whilst the pressure at the centre is supposed to differ but little from the normal gas pressure.

Consider the case of two long strips, one narrower than the other, but both at the same distance from the glass wall, and also both 1°C. above its temperature. If the excess pressure is restricted to the edges, we should expect the narrower strip

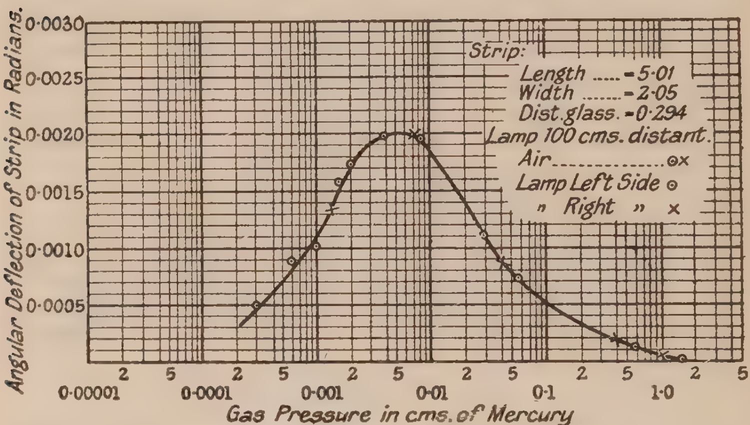

FIG. 7.—DEFLECTIONS OF COPPER STRIP NO. II. IN CYLINDRICAL BRASS BOX. Graphical Record of Results.

to be deflected most. If, however, the pressure is uniform, the deflections will be the same, whilst if the pressure increases from the edge to the centre, the larger strip will be deflected most.

In the present experiment, two copper foil strips, widths 2 cm. and 3 cm. respectively, were placed at a distance of 0·3 cm. from the glass wall of the box. The distance 0·3 cm. could be considered small compared to the width of either strip, and thus at a given pressure, the temperature rises of the strips under the influence of radiation from a lamp a definite distance away, could be considered approximately the same.

The results are shown in Figs. 6 and 7. It will be seen that the wider strip is deflected most at all pressures. It is possible that a certain amount of this difference may be accounted for by the slightly higher rise of temperature of the wider strip,* but it is probable that the chief difference is accounted for by a pressure which rises in value from the edges, quickly attains a maximum value, and remains constant over the greater part of the strip.

Similarly corresponding curves were obtained when the box was filled with hydrogen.

Experiment IV.—It had been noticed that the deflection of the strip depended on its distance from the glass wall. With a view therefore to obtaining more definite information, a

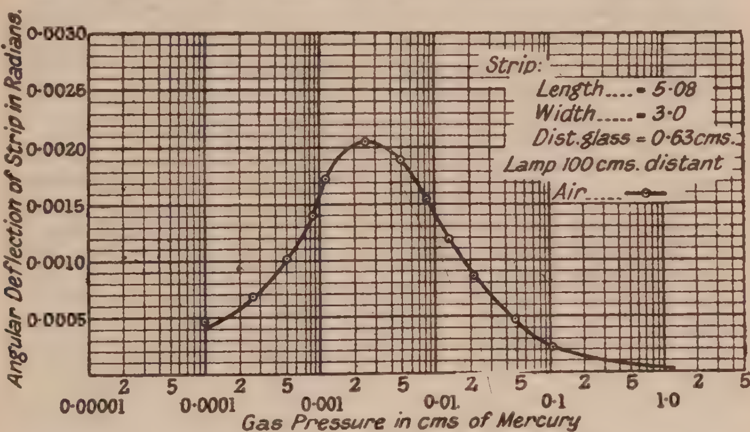

FIG. 8.—DEFLECTIONS OF COPPER STRIP III. IN CYLINDRICAL BRASS BOX. Graphical Record of Results.

series of readings was taken of the deflections of the copper strip at about twice its previous distance. As shown by Fig. 8, all the deflections observed are smaller, but as the temperature to which the strip rises in this case is necessarily higher, the meaning of the present results in relation to the previous ones, requires to be made clear by a reduction of both sets to deflections per 1°C. rise of temperature of the strip. Such a reduction will be referred to later.

* The formulæ of Kirchhoff (Winkelmann, "Handbuch der Physik," IV., pp. 35-36) and Maxwell ("Elec. and Mag.," Vol. I., p. 286) are apparently not of much use in helping to give a better approximation to the true temperature rise.

Experiment V.—A larger amount of heat is conducted from the edges of the strip than from the central portions, and a temperature gradient, whose value depends on the thickness of the strip, thus exists. It was thought that this temperature gradient might have something to do with the production of the pressure on the strip, and with a view to varying the temperature gradient therefore, experiments were carried out with foils ranging in thickness from about 100×10^{-5} cm. to $0 \cdot 9 \times 10^{-5}$ cm. In the former case a sufficient difference of temperature was produced between the two surfaces to give deflections that depended on the side on which the lamp was placed, to the extent of as much as 20 per cent. In the following table of results calculated from curves, the mean of these two values has alone been recorded. It should be noted further that, owing to their smallness, the deflections could not, in this case, be observed very accurately.

Gas pressure cm. mercury.	Ratios of forces on strips, calculated from strip deflections (lamp same distance throughout).		
	Gold strip.	Copper strip.	Aluminium strip.
	Thickness $= 0 \cdot 9 \times 10^{-5}$ cm.	$0 \cdot 4 \times 10^{-4}$ cm.	$0 \cdot 1 \times 10^{-2}$ cm.
	Thermal conductivity $= 0 \cdot 70$.	0·92	0·5
0·1	1	1·03	...
0·03	1	1·20	...
0·01	1	1·19	1·13
0·003	1	1·19	1·44
0·001	1	1·03	1·39
0·0003	1	0·98	1·37

It was thought probable that the differences in the forces acting on the strips could be satisfactorily accounted for by the different amounts of radiation absorbed, and hence by the different temperature rises. Measurements were accordingly made of the relative absorbing powers of gold, copper and aluminium foil, by observing, with a thermo-junction, the initial rate of rise of temperature on exposure to the radiation, of a copper block, one of whose faces had been covered with the foil in question. The method has certain disadvantages, for the absorbing power of a metal surface is very small, and care has to be taken to avoid heating of the block by warm air currents produced in the vessel, in which the block is contained. Moreover, the surface of the thin foil may suffer a change when it is stuck on the surface of the block. The

following values, however, were obtained for the relative absorbing powers :—

Gold	$=1.$
Copper	$=0{\cdot}98.$
Aluminium	$=1{\cdot}41.$

It would seem therefore as if the differences in the forces acting on the various strips could be accounted for chiefly by the different amounts of radiation absorbed, and hence by the different temperature rises. The temperature gradient in the strip from the edge to the centre would, moreover, seem to have had but little influence in the production of the pressure on the strip.

Experiment VI.—It had been noticed that if the front glass of the box containing the lamp was removed, and the light switched on for a considerable period, the strip did not return to the original zero. By filtering the radiation through one or more plates of glass cut from the same sheet as the window of the box, however, it was possible to reduce the creeping of the zero to a very small value.

The phenomena connected with the repulsion of a metal strip from a hot wall appeared to be worth investigation, and accordingly the following experiment was planned :—

At a distance of about 0·6 cm. from the glass wall a large sheet of aluminium foil 0·001 cm. thick covering nearly the whole of the front face of the box was fixed by means of silk threads attached to small glass blocks. Attached in turn to the aluminium foil was a fine horizontally-placed fibre of glass or silk carrying the strip of aluminium foil. The first strip used was 4·6 cm. long and 3 cm. broad and $0{\cdot}67 \times 10^{-4}$ cm. thick, and was placed first 0·6 cm. from the aluminium screen and later 0·3 cm. The second strip used was of the same thickness, but was 2·5 cm. long and 1·0 cm. broad, and was placed approximately 0·1 cm. from the screen. The whole arrangement is represented in Fig. 3. Radiation from the lamp falls on the screen and raises its temperature, and, if the gas pressure is low enough, a deflection of the thin aluminium strip can be observed in the usual way. A series of results was thus obtained, but like those of Experiment IV., they are more easily considered when reduced so as to show the force per square centimetre of strip per 1°C. rise of temperature of the screen. The reduced results are shown in Fig. 10.

Experiment VII.—Instead of mounting the strips parallel to the glass wall, the effect was tried of mounting a strip 0·6 cm. wide at an angle of about 45°, the nearest edge of the strip being about 0·1 cm. from the glass. A diagram of the arrangement is given in Fig. 4. Another strip was similarly mounted at an angle of about 45° to the screen. When intense radiation was concentrated on the strip near the glass wall by means of a lens, it was found that, over a considerable range of pressure near 0·01 cm. mercury, the strip, instead of being deflected away from the glass wall, would sometimes move edgewise towards it, and remain there until the radiation was cut off. If the nearest edge of the strip was more than about 0·1 cm. from the glass wall a slight mechanical shock was generally necessary before the edgewise movement would take place. When the radiation was concentrated on the screen (the lamp being on the side remote from the strip) the strip was always repelled, and the edgewise movement could never be induced.

Experiment VIII.—A strip about 0·6 cm. wide and about 3·5 cm. long was placed symmetrically in a small glass box 1 cm. wide, 2·5 cm. long and 4 cm. high. A plan of the arrangement is shown in Fig. 5. When light from the lamp was concentrated on the strip by means of a lens, and a small mechanical shock administered to the table on which the box rested, the strip moved edgewise to one or other face of the box, and remained there until the radiation was cut off. The action was most vigorous at a pressure of about 0·01 cm. mercury, but extended over a considerable range.

Reduction of Curves.

The results of Experiments III., IV. and VI. can be rendered much clearer by reducing them to the force per square centimetre of strip or screen per 1°C. rise of its temperature. Before any such reduction can be made, however, we require to know the way in which the temperature of the strip or screen varies as the gas pressure is reduced—the distance of the lamp, of course, being kept constant.

Reduction of gas pressure diminishes the effective conductivity of the gas, and although at first the diminution is small, it becomes marked when the mean free path of the molecules reaches an appreciable fraction of the distance of

the strip or screen to the glass wall. Thus strip or screen temperatures rise with reduction of gas pressure.

To measure the temperature rises of the strips, several experimental methods were tried, but eventually the thin strip of metal leaf was replaced by foil of sufficient thickness to admit of the attachment of a very fine thermo-junction. The method gave consistent results, but there was some doubt as to what extent they corresponded with the actual temperature rises of the original strips.

The problem was therefore approached from a different point of view. By coating the surface of a copper block with the foil in question, and by measuring the initial rate of rise of temperature when the surface was exposed to radiation from the lamp, an estimate of the quantity of energy Q absorbed per square centimetre per second by the strip was obtained. By equating Q to the heat lost from the strip by conduction through the gas and by radiation, the author found that, as a first approximation, T, the temperature rise of the strip could be written

$$T=Q\bigg/\left[k\left\{\frac{1}{d_1+2c\lambda}+\frac{1}{d_2+2c\lambda}\right\}+2R\right],$$

where k is the conductivity of the gas, d_1 and d_2 the distances from the glass walls, R the emissivity, λ the mean free path of the molecules, and where c is a constant depending on the nature of the gas, and whose significance will be discussed later.

A correction was also applied for the conduction of heat through the supports of the strip.

Although the absolute values differ somewhat, the fractional variations of the temperature rises of the strips with gas pressure, obtained by this and by the thermo-junction method, are in good agreement as low as 0·01 cm. mercury, but after this gas pressure, the former values are markedly in excess. It is the values given by the calculation that have been used in the reduction of the curves, although it is admitted that uncertainties in the values of c and R may introduce serious errors.

So far as the immediate purpose of the present work is concerned, however, no very accurate results are required. It is rather qualitative guidance that is sought, and the reduced curves, which are shown in Figs. 9 and 10, should be regarded in this light.

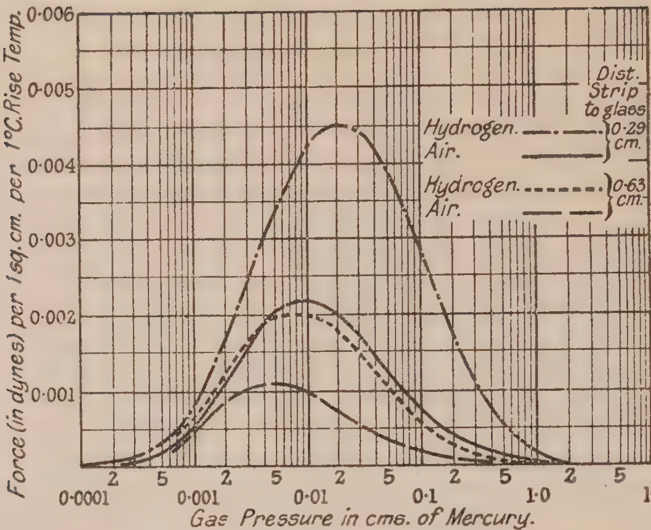

FIG. 9.—RELATION BETWEEN FORCE ON 1 SQ. CM. OF COPPER STRIP III. PER 1°C. RISE OF TEMPERATURE, AND GAS PRESSURE.

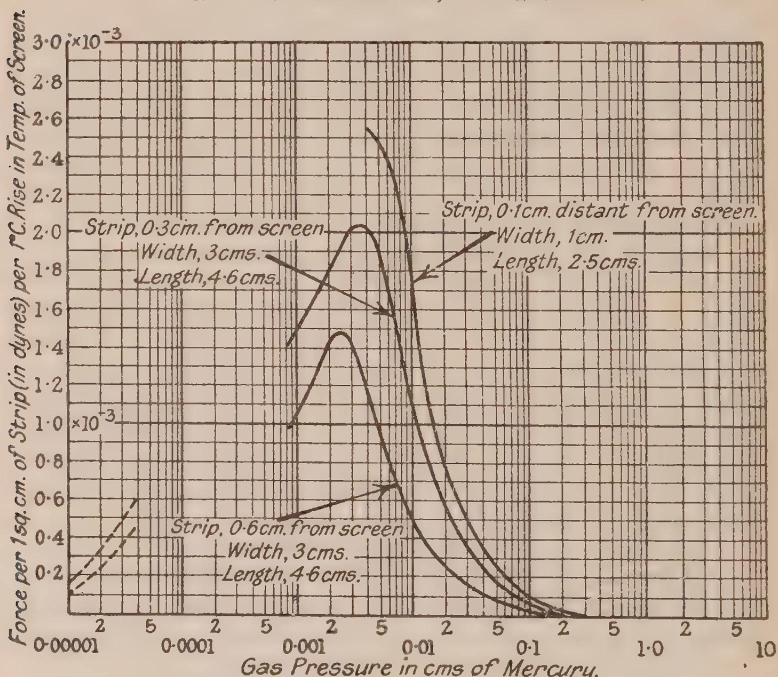

FIG. 10.—RELATION BETWEEN FORCE PER 1 SQ. CM. OF STRIP PER 1°C. RISE IN TEMPERATURE OF SCREEN, AND GAS PRESSURE.
Aluminium Strips VIII. and IX.

Theoretical Discussion of Results.

It is natural to attribute the movements of a light, freely suspended, heated body, to convection currents in the surrounding gas. Convection currents, however, in any apparatus depend fundamentally on the ability of a small volume of air to maintain for some time its high temperature and hence its buoyancy. If the mean free path of the molecules be sufficiently large, it is clear that they will drive their way through any such small volume, and that a uniform condidition will soon be established. Thus, with reduction of pressure and increase of mean free path, convection currents tend to disappear.*

In Experiment I. it will be noticed that from atmospheric pressure down to about 1 cm. of mercury, two movements of the strip are clearly superposed. The more slowly established movement dies out as the pressure is reduced, whereas the quickly established movement increases with reduction of pressure. It has been thought reasonable in this, as in previous work,† to attribute the former movement to convection currents, whilst the latter movement has been attributed to forces about whose nature a theory is now to be put forward. The theory is based on a previous Paper by the author‡ on thermal transpiration, and is apparently capable of explaining all the present experimental facts.

It will be remembered that in the Paper referred to it was shown that if a temperature gradient were maintained along a tube of uniform bore, the walls of the tube, in forcing the gas to take their temperature, exerted a traction upon the gas, and caused it to flow from the cold to the hot side. With finite spaces at the ends of the tube, it was shown also that a

* In this connection it should be noted that rapidly moving molecules, such as those of hydrogen, are of necessity more effective in removing temperature inequalities than, for instance, the more slowly moving molecules of air. The difficulty of maintaining convection currents in hydrogen is now well known.

† "Proc." Phys. Soc., XXVIII., p. 259, 1916.

‡ "Proc." Phys. Soc., XXXI., p. 278, 1919. As frequent references are made to this Paper, the author would like to take the opportunity of making the following errata :—

P. 281, fourth and fifth lines from bottom, for $-\frac{1}{2}\frac{d\Omega}{dr}\lambda$ read $+\frac{1}{2}\frac{d\Omega}{dx}\lambda$.

P. 285, fourteenth line from top, for $\frac{2k}{T}\frac{dT}{dx}$ read $\frac{k}{T}\frac{dT}{dx}$.

P. 287, ninth line from bottom, for 1 cm. read 0·1 cm.

sufficient pressure was developed on the hot side to cause an equal flow, of the Poiseuille type, in the reverse direction.

The result of the superposition of the two flows was to give a gas current near the surface of the tube from the cold side to the hot side, and a current in the reverse direction along the axis, whilst between the two there was a surface of zero velocity. With the reduction of gas pressure the Poiseuille counter-flow became less and less, until at the highest rarefactions, it was quite unimportant, and the hot regions were enabled to maintain undiminished their higher pressures.

In this latter stage, the pressure temperature gradient was found proportional to the pressure of the gas, but independent of its nature. At high pressures, however, when the mean free path of the molecules was small compared to the diameter of the tube, it was found to be inversely proportional to the gas pressure, and to be greater for hydrogen than for air.

The theory was worked out on conditions that were much simpler than those that obtain here, and it is only proposed in the present instance to look to the theory for general guidance, and not for quantitative agreements. In this, as in the previous work, however, a flow of gas is supposed to take place from the cold to the hot regions, but as the extent of a "hot region" changes with variation of gas pressure, some preliminary considerations are necessary.

Consider first of all the case of two parallel plates—one hot and the other cold. We may regard the molecules that strike the hot surface not only as coming on an average from a distance λ, the mean free path of the molecules, but also as possessing the mean temperature of this region. If, after reflection at the surface, these molecules merely acquire its temperature, it is clear that the mean temperature of the surface layer of gas is necessarily lower than that of the surface itself. A surface temperature discontinuity thus exists, which becomes more marked the larger the value of λ.

One result of the discontinuity is that the quantity of heat passing between the plates is reduced, and it is now customary to regard this reduction as due to a change in the surface conditions, and not to a change in the internal conductivity of the gas itself. Smolan,* Warburg and Gehrcke,† and Lasareff‡ have in fact shown that the quantity of heat conducted per

* "Akad. Wiss. Wien. S. Ber.," CVIII., pp. 5-23, 1899.
† "Ann. d. Phys.," II., 1, pp. 102-114, 1900.
‡ "Ann. d. Phys.," XXXVII., 2, pp. 233-246, 1912.

second between the two unit surfaces distant d apart and differing in temperature by 1°C. may be expressed as $\frac{k}{d+2c\lambda}$, where k is the ordinary conductivity of the gas. The effective distance between the plates must thus be increased by $2c\lambda$.

If a molecule reached thermal equilibrium on collision, the value of c would be unity. Many observers besides those mentioned, and including Soddy and Berry,* Knudsen,† Smoluchowski,‡ and Langmuir,§ have demonstrated, however, that this is not the case, and have assigned values to c. Smoluchowski has pointed out that the interchange of energy between colliding spheres is the more imperfect the greater the difference of their masses, and we might thus expect the temperature discontinuity to be greater for the light than

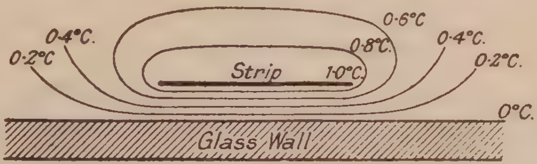

FIG. 11.—ISOTHERMS. MEAN FREE PATH SMALL COMPARED WITH DISTANCE OF STRIP TO GLASS WALL.
Rise of Temperature of Strip=1°C.

for the heavy gases. This is so, but Langmuir and Knudsen have shown that the whole problem is much modified by the presence of surface layers of condensed gas. Support is lent to their conclusions by the fact that at a given pressure, c varies but little with the nature of the surface, provided only that the latter is polished.

According to Warburg, c varies somewhat with gas pressure, but for the present work it may be taken as constant and about 5·8 for hydrogen and 1·65 for air.

Let us now turn our attention to the case of the metal strip suspended parallel to the glass wall, as in Fig. 2. The shapes of the isothermals near the strip and the closest wall are, at high pressures, somewhat as represented in Fig. 11. As, however, the gas pressure is lowered, the high temperature isothermals tend to disappear as a result of the temperature

* "Proc.," Royal Soc. A LXXXIV., p. 576, 1911.
† "Ann. d. Phys.," XXXIV., 4, p. 593, 1911.
‡ "Phil. Mag.," XXI., p. 11, 1911.
§ "Phys. Rev.," Vol. II., No. 5, p. 329, 1913.

discontinuity, and we are confronted with a system of isothermals roughly sketched in Fig. 12. Finally, when the mean free path is long compared to the dimensions of the vessel, there will be hardly any temperature gradient from the strip to the opposite plate. With infinite plates, the whole of the gas would be at a temperature half way between that of the hot and cold plates, but in the present instance the temperature will vary slightly from point to point. The temperature near the edges of the strip for instance, would be somewhat lower than near the middle of the strip.

At medium pressures, therefore, any plane placed parallel to the strip and near to it, will coincide with the isothermals in the central portions, but will intersect successive iso-

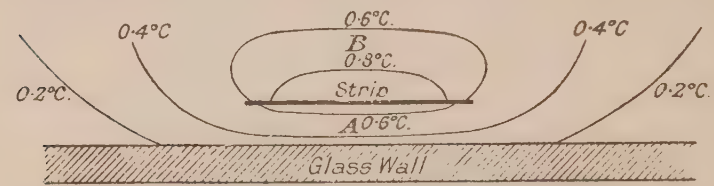

FIG. 12.—ISOTHERMS. PRESSURE SUCH THAT $c\lambda$=DISTANCE OF STRIP TO GLASS WALL.

Rise of Temperature of Strip=1°C.

thermals in the regions near the edges. Similar considerations apply to a plane placed close to the nearest wall. We have, in fact, both in the gas layer at the edges of the strip and in the gas layer on the opposite wall, a definite temperature gradient. Although this temperature gradient is not uniform in character, and although it does not extend directly across from strip to plate, yet there is a certain resemblance to the conditions of the previous Paper,* and it has been thought a legitimate step to apply in a very general way the results there obtained.

If this plan be adopted, we must assume that on the two sides of the strip and on the opposite wall, we have a flow of gas from the exterior to the interior, together with an equal central counter-flow from the interior to the exterior. The superposition of these two flows might thus quite well be represented by a diagram such as Fig. 13. Over a wide region near A there will be little motion, and the pressure will be sensibly constant, whilst near the edges the pressure will fall

* "Proc." Phys. Soc., XXXI., p. 278, 1919.

off gradually—partly as the result of the existence of motion, and partly as the result of the flow on the other surface of the strip. This latter flow, of course, produces a pressure in opposition to that on the side A and when the strip is placed symmetrically in any vessel, the resultant pressure is in theory zero, and in practice negligible. In fact, strips mounted thus, were previously used for the estimation of the pressure of light.

It is thus possible to explain why in Experiment II. the deflections of the strip do not depend on a difference of temperature of the surfaces, and also why in Experiment III. the pressures do not appear as " edge effects."

The results of Experiment V. with thicker strips, likewise find an easy explanation in the present theory. We should

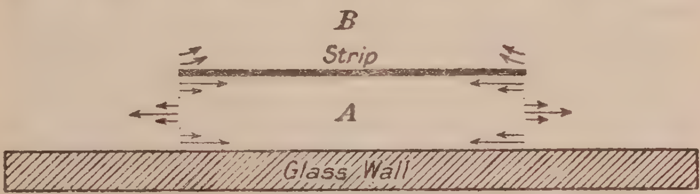

FIG. 13.—FLOW OF GAS ROUND HEATED STRIP PLACED IN FRONT OF COLD GLASS WALL.

expect the temperature gradient due to the temperature discontinuity, far to outshadow in importance the small temperature gradient on the strip itself, due to a greater quantity of heat being conducted away from its edges. It is the temperature of the layer of gas that is important, not that of the material of the strip.

In addition, it may be noted that the force per square centimetre on the strip is greater for hydrogen than for air. According to the previous theory, the ratio should be

$$\eta^2_h/\rho_h : \eta^2_a/\rho_a = 3{\cdot}4 : 1,$$

where η and ρ refer to viscosities and densities respectively. The ratios in both the experimental curves are roughly of this order, although no exact agreement can be expected.

Other points of agreement may be noted. The pressure at which a maximum value is obtained for the resultant force on the strip, is lower for air than for hydrogen, both according to theory and to experiment. Further, as in Experiment IV., the maximum value occurs at a lower pressure when the dis-

tance of the strip from the glass wall is greater. This corresponds in the previous work with the widening of the bore of the tube. It is easily seen that such a widening results in shifting the maximum of the curves to the left.

As we proceed to still lower pressures, questions of gas flow become far less important, and we are then concerned instead with the mean temperature of the two regions on either side of the plate. As their limiting values are very close, the resultant pressure on the strip soon begins to fall to a very low figure. It should be noted, moreover, that the values both for hydrogen and for air approach each other, and that the pressure on the strip is now roughly proportional (whereas before it was inversely proportional), to the gas pressure. This is all in accord with the conclusions of the previous Paper.

Let us now pass to the case where the strip is suspended, as in Fig. 3, in front of a screen heated by radiation. We are confronted here with quite a different system of isothermals. At high pressures they are, with the exception of the region near the edges of the screen, parallel planes, and the strip will acquire the temperature of the isothermal with which it coincides. We must, however, associate with the rise of temperature of the screen, a flow system similar to that round the strip in Fig. 13. Notwithstanding this a strip suspended parallel to the screen as in Fig. 3 will be out of the region in which the flow occurs, and it will not be affected. An inspection of the curves of Fig. 9 reveals the fact that at a gas pressure of 0·1 cm., for instance, there must, by analogy with the case of the strip, be a very considerable gas flow round the edges of the screen. Nevertheless the deflections of the strip are very small, as shown in Fig. 10.

With decrease of gas pressure, the previously inconsiderable temperature discontinuities at the surfaces, have to be taken into account, together with the resultant changes of form of the isothermals. Let us first suppose the screen to be a sufficiently good conductor to maintain itself at a uniform temperature. Then, in addition to the temperature discontinuities at the screen and at the glass wall, there will be others at the surfaces of the strip, and thus in effect the distance from the screen to the opposite cold wall will be increased by $2c\lambda$. A somewhat smaller quantity of heat thus passes from the portion of the screen opposite the strip, and the temperature gradient is likewise smaller. An attempt has been made to

embody these facts in the rough drawing of the isotherms given in Fig. 14.

As the pressure is still further reduced, region A will approximate to a mean temperature three-quarters that of the screen in excess of the temperature of the walls, whereas region B will approximate to a mean excess temperature one-quarter that of the screen. These facts are easily understood when we remember that, since the effective distances from screen to strip and strip to glass wall are both $2c\lambda$, the temperature excess of the strip must be half that of the screen.

In practice the screens used did not possess a sufficiently great conducting power to maintain a perfectly uniform temperature. To calculate the temperature distribution over

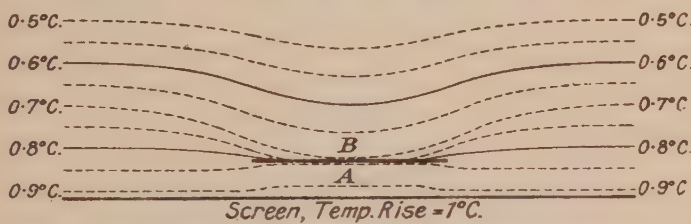

FIG. 14.—ISOTHERMS. GAS PRESSURE SUCH THAT $c\lambda$ = DISTANCE STRIP TO SCREEN.

their surfaces, making allowances for the relative conductivities of the screens, the strips and the surrounding gas, would be a difficult problem into whose solution it is unnecessary to go for the purposes of the present Paper. It is here quite sufficient to proceed to the other extreme limit, and to assume the screen a non-conductor, and to work out the temperature of the strip on the supposition that a quantity of heat, sufficient to raise the temperature of a portion of the screen remote from the strip 1°C., is transmitted directly across from screen to strip. Applying this assumption to the central portions of the strip we are led to suppose that the isotherms are somewhat as represented in Fig. 15. It will be noticed that there are some easily explicable differences between these isotherms and those of Fig. 14.

At the highest rarefactions the mean temperatures of regions A and B can be easily obtained if we remember that in effect, the distances from glass to screen, and from screen to strip are all $2c\lambda$. If, therefore, all the energy that passes from screen to strip passes on directly to the opposite glass

wall, it follows that the temperature excess T of the strip must be half the temperature excess of the screen. Now, if the strip were absent, the amount of energy required to raise the screen 1°C. would be $k/c\lambda$, since heat is lost from both sides of the screen. Equating therefore $k/c\lambda$ to the quantity of heat transmitted from the screen to the near glass wall, plus the quantity transmitted from screen to strip, we have

$$\frac{k}{c\lambda}=\frac{2Tk}{2c\lambda}+\frac{Tk}{2c\lambda}$$

whence

$$T=\tfrac{2}{3}.$$

The temperature excess of the screen is thus $1\frac{1}{3}$°C., whilst that of the strip is $\frac{2}{3}$°C. Thus, further, the mean temperature

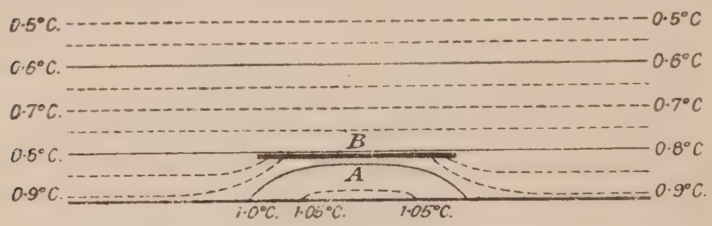

FIG. 15.—ISOTHERMS. GAS PRESSURE SUCH THAT $c\lambda$=DISTANCE STRIP TO SCREEN.

Rise of Temperature of Parts of Screen Remote from Strip=1°C. Energy conducted away from all Parts of screen is the same.

excess of region B above the temperature of the walls, will be 1°C., whilst that of region A will be $\frac{1}{3}$°C.

In practice we must suppose that the actual isotherms lie somewhere between those associated with the limiting cases described above. These isotherms change with reduction of pressure in a way quite different to those in Fig. 12, a noticeable difference being the fact that in the present case the mean temperature difference between the exterior and the interior increases progressively. Now, in the previous Paper, it was found that, at high pressures, a constant difference of temperature between various regions was associated with a thermal transpiration pressure varying rather more slowly than the inverse of the gas pressure. Hence, with an increasing temperature difference, as in the present instance, we should expect a more rapid variation. Such a variation is exhibited in the curves of Fig. 10.

It should be noted that, as a contrast to the previous case, the pressures on the two sides of the strip do not act in opposition. On the contrary, they co-operate, and we must, at high pressures, represent our systems of flow somewhat as in Fig. 16. It will be seen that the currents reduce the pressure in the region B, and increase it in the region A. At the very lowest pressures, moreover, the co-operation still exists, the mean temperature of A being above that of the exterior, and the mean temperature of B being below that of the exterior. If we make use of a result established in the previous Paper that when the mean free path of the molecules is very great, the pressures in various parts of an enclosure are proportional to the square roots of their absolute temperatures, we can easily calculate the resultant pressure on the strip. In fact, if ΔT be the rise of temperature of the screen, it can be easily

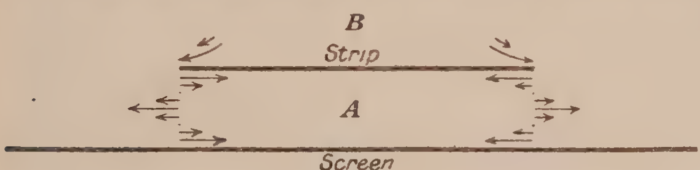

FIG. 16.—FLOW OF GAS ROUND STRIP PLACED IN FRONT OF HEATED SCREEN.

shown that the pressure on the strip is given by $\Delta Tp/4T$, where T is the absolute temperature, and p the pressure of the gas in dynes. The dotted lines in Fig. 9 give values calculated from this formula for the two limiting cases previously described, and these lines appear as not improbable asymptotic limits of the experimental curves.

It may be remarked that the formula $\Delta Tp/4T$ was first obtained and verified by the Danish physicist Knudsen for pressures so low that the mean free path of the molecules was large compared to the distance of the strip to the heated walls. The present work, of course, deals chiefly with much higher pressures.

The theory hitherto employed postulates among other things a flow of gas from the exterior, along the surface of the strip. The assumption of such a flow is supported by the apparently satisfactory general explanations that can be given of the experimental results. More direct evidence can, however, be obtained. The gas can only flow as a result of an equal and opposite reaction on the material of the

strip. In the case of a strip mounted parallel to the glass, as in Fig. 2, the two reactions on the opposite edges balance. If, however, we destroy this symmetry by mounting the strip at an angle to the glass, as in Fig. 4, there is the possibility that the unbalanced reaction on the strip might be of more importance than the repulsion from the glass, and thus cause the strip to move edgewise to the glass. This, of course, actually happens. When, however, the light is concentrated on the screen and not on the strip, as in Experiment VII., the repulsion effect is apparently too strong to permit such edgewise movement.

A further proof of the existence of the reaction on the strip is afforded by Experiment VIII. Here the reactions are in unstable equilibrium. If, as the result of a small mechanical shock, the symmetrical placing of the strip is disturbed, a greater temperature gradient is set up near one edge, and hence a greater reaction. This results in the strip again moving edgewise to the glass.

Summarising, therefore, it may be said that the experiments establish the fact that, at gas pressures below 1 cm. mercury, the movements of the strips of foil mounted in the various ways described, result neither from convection currents, nor from a difference of temperature of their surfaces, nor from "edge pressures." A previous Paper by the author on thermal transpiration, provides however an apparently satisfactory basis for the explanation of these results, and for others relating to variations of the deflections of the strips with the nature of the surrounding gas and with its pressure. Whilst at the lowest pressures conditions are comparatively simple, at the higher pressures they are complicated by gas currents of the thermal transpiration type. For the existence of such gas currents experimental evidence can be given.

The author has pleasure in thanking Prof. Lees for the facilities afforded for the prosecution of this research. The author has also to thank three of his students, Messrs. Andrews, Gregson and Harling, for the preparation of numerous curves and diagrams.

Abstract.

The present Paper arises out of two previous Papers by the author on the pressure of light ("Proc." Phys. Soc., XXV., p. 324, 1913, and XXVIII., p. 259, 1916), and consists of an experimental investigation of the nature of certain peculiar movements of strips of thin metal foil surrounded by rarefied gases, and exposed to radiation. The experiments deal

chiefly with phenomena at gas pressures below 1 cm. of mercury, and it is shown that the apparently diverse results obtained can be connected by a theory based on the work of a previous Paper (" Proc." Phys. Soc., XXXI., p. 278, 1919).

The author concludes that, at the highest rarefactions, the pressures on the strips arise from the fact that, if differences of temperature exist in an enclosure, the pressure of the gas is not uniform, but varies approximately as the square root of the latter's absolute temperature. The simple conditions that exist at low gas pressures are complicated, at the higher pressures, by gas currents which differ fundamentally from convection currents, but which are closely connected with the phenomena of thermal transpiration.

DISCUSSION.

Prof. Eccles said he was indebted to the author for his elucidation of gas action effects. He had frequently been puzzled by them in the past when designing certain instruments. In the curves shown the maximum effects occurred at a pressure of about $\frac{1}{100}$th cm. of mercury. What ratio did the mean free path of the gas at this pressure bear to the distance between the foil and the glass plate ? Had this ratio any particular bearing on the fact of the gas action being a maximum ?

Mr. West said the ratio was about one-sixth or one-fifth, though he could not be quite certain without reference.

Mr. F. E. Smith said that the author had experimented with two widths of foil. He suggested that experiments should be made with a series of strips of different widths and a curve obtained connecting the width of the strip and the deflection.

Mr. West said he had done this, but, except for broad strips, it was impossible to be certain that the temperatures were the same, as the disposition of the stream lines was different. It was necessary, therefore, to reduce the results to deflections per degree rise of temperature of the strip. At present his method of reduction was only rough, but the results showed that if the width of the strip is not comparable with the mean free path, the effect decreases as the width is diminished.

Mr. F. J. W. Whipple mentioned that when the author read his last Paper, Mr. Lewis Richardson also read one describing a new manometer. He gave a diagram connecting the force on the circular diaphragm of his instrument with the pressure, and afterwards he (Mr. Whipple) had compared Mr. West's transpiration theory with Richardson's diagram. According to the theory, the annular channel in the manometer was equivalent to a tube of 7 cm. diameter, which agreed very closely with the actual dimensions of the annulus. This appeared to afford confirmation both of Mr. West's theory and of Mr. Richardson's measurements.

The Author, in reply to Prof. Eccles, communicated that the maxima in the reduced curves corresponded to a mean free path somewhat greater than one-quarter the distance of the strip to the glass wall. An indication of the relations of the various maxima to the mean free path was given by the theory of the previous Paper. In reply to Mr. Smith, the author stated that experiments with narrower strips had been tried. Unlike the broad strips, however, the rise of temperature could not be assumed independent of the width, as the disposition of the stream lines was different. It was necessary, therefore, to reduce the results to deflections per degree rise of temperature of the strip. At present the method of reduction was only rough; but, as far as they went, the results indicated that the effect decreased as the width was diminished. The previous results were thus confirmed.

XIX. *Absorption of Gases in the Electric Discharge Tube.* By F. H. Newman, *A.R.C.Sc., B.Sc., University College Exeter.*

Received January 8, 1920.

1. *Introduction.*

The following experiments dealing with the absorption of gases in the discharge tube were made in continuation of those described in "Proceedings" Royal Society A., Vol. XC., 1914 It was shown that both hydrogen and nitrogen are absorbed in the discharge tube when sodium—potassium alloy is used as the electrode—whether cathode or anode. Also the amount of gas absorbed compared with the quantity of electricity passing in the secondary circuit was determined. It was found that the absorption increased as the pressure decreased, and was greater when the alloy was made the cathode than when it was the anode.

The alloy has properties similar to those of potassium and sodium, and so one would expect that these metals when used as electrodes in the discharge tube would also absorb nitrogen. As regards hydrogen, absorption cannot take place, as with this gas a scum appears at the surface of the alloy. By shaking the tube this scum can be removed, but in the case of the solid metals this cannot be done. The experiments now described were made to see if the metals absorb nitrogen, and if so how much gas is absorbed for different quantities of electricity passing in the secondary circuit—the measurements being made with various pressures of the gas. In order to measure the electricity, a small voltameter was employed, and the quantity of hydrogen gas liberated in it noted during a reading. The secondary discharge was made unidirectional by employing a cathodic valve in the secondary circuit of an induction coil.

2. *Preliminary Experiments.*

Fig. 1 shows the apparatus used and was similar to that employed in the previous experiments. The nitrogen was prepared in A by allowing air to stand over phosphorus for several hours. The gas then passed over phosphorus pentoxide in B for drying, and could be admitted into C—which was a known volume enclosed between two taps T_1 and T_2—by opening T_1. D was an oil manometer and E the discharge tube—with platinum wire electrodes—connected to a Toepler

pump for exhaustion. In the experiments the apparatus was first exhausted, and the manometer calibrated as follows: Opening T_1 and then closing it a known volume of gas practically at atmospheric pressure was enclosed between T_1 and T_2. Opening T_2, the gas filled the apparatus and the alteration in pressure which it produced could be read from the manometer. By allowing several volumes to enter in this way, and noting the pressure before and after, the amount of gas which caused a recorded difference of pressure was found. So

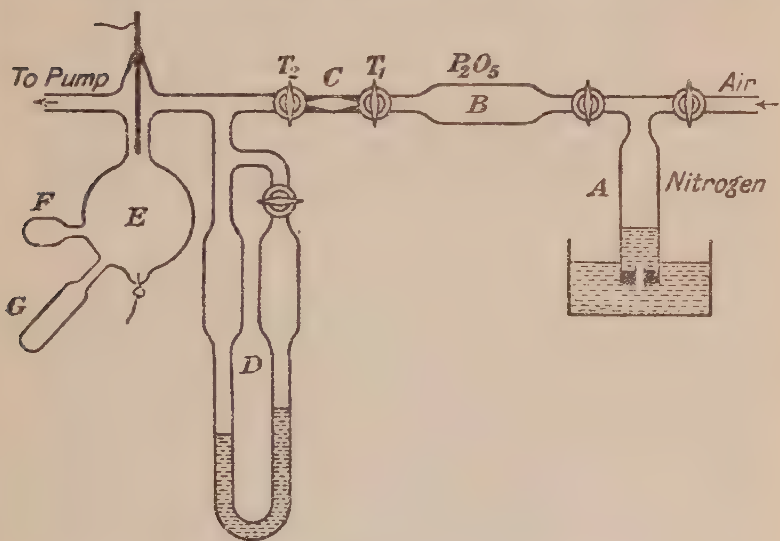

FIG 1.

that when absorption took place, from the initial and final readings of the manometer the amount of gas at atmospheric pressure which had been absorbed was obtained. First sodium was tried. It was placed in a side tube F and on heating the latter the metal melted and ran down to the bottom of E, covering the platinum wire electrode. It was found, however, that no absorption took place under these conditions, owing to the impurity of the metal. To overcome this difficulty the discharge tube was silvered inside and sodium in a side tube G was distilled into E in a vacuum. By this means over the surface of the silver a layer of pure sodium was obtained. The same method was employed later with potassium.

3. *Absorption of Nitrogen by Sodium.*

Observations were made with sodium as the cathode and later as the anode in the discharge tube. In both cases there was absorption of nitrogen, but the amount absorbed was greater when the metal was the cathode. The absorption increased as the pressure decreased, and although the amount of gas absorbed was many times greater than that absorbed by the walls of the vessel during discharge, yet it was not as great as when the sodium-potassium alloy was used. There appeared to be no falling off of the absorption with time.

4. *Absorption of Nitrogen by Potassium.*

Absorption also took place with potassium, both when used as the cathode and anode. In this case, however, the amount of nitrogen absorbed compared with the quantity of hydrogen liberated in the voltameter was smaller than when sodium was used.

5. *Effect of Heat on Absorption by the Sodium-Potassium Alloy.*

It has been shown in the previous Paper that the absorption of the gas by the alloy is probably a chemical action resulting in the formation of the nitrides of the metals and not an occlusion of the gas in the alloy. To strengthen this view the discharge tube was heated by means of an electric heater, and the amount of gas absorbed compared with the quantity of electricity passing in the circuit determined at different temperatures. It was found that when the alloy was used as cathode or anode, there was no difference in the rate of absorption from 50°C. to 200°C., but above this temperature the rate of absorption increased. Also when the discharge was stopped and the alloy heated to 300°C., no liberation of the nitrogen took place, so that the nitrides formed must be fairly stable.

6. *Tabulation of Results.*

A few of the results obtained are shown in the following tables, and for reference the amount of nitrogen absorbed in the case of the sodium-potassium alloy has been shown. The current in the secondary varied from 5 to 10 milliamperes, and the results were independent of the shape and size of the discharge tube.

TABLE I.—*Absorption of Nitrogen by Sodium.*

Sodium used as cathode.					Sodium used as anode.				
Volume of N_2 absorbed (A)	Pressure	Volume of H_2 liberated in voltameter. (B)	Value A/B.	Value A/B at same pressure for $Na-K$ alloy.	Volume of N_2 absorbed (A)	Pressure	Volume of H_2 liberated in voltameter. (B)	Value A/B.	Value A/B at same pressure for $Na-K$ alloy.
c.c.	mm.Hg.	c.c.			c.c.	mm.Hg.	c.c.		
0·36	8·1	0·52	0·7	2·3	0·27	6·5	1·63	0·2	0·7
0·18	6·6	0·21	0·9	2·3	0·54	5·1	2·70	0·2	0·9
0·54	5·5	0·51	1·1	2·4	0·19	3·7	0·53	0·4	1·4
0·35	3·4	0·30	1·2	2·6	0·37	2·2	0·72	0·5	1·9
0·55	1·0	0·42	1·3	2·7	0·36	0·7	0·60	0·6	2·5

TABLE II.—*Absorption of Nitrogen by Potassium.*

Potassium used as cathode.					Potassium used as anode.				
Volume of N_2 absorbed (A)	Pressure	Volume of H_2 liberated in voltameter. (B	Value A/B.	Value A/B at same pressure for $Na-K$ alloy.	Volume of N_2 absorbed (A)	Pressure	Volume of H_2 liberated in voltameter. (B)	Value A/B.	Value A/B at same pressure for $Na-K$ alloy.
c.c.	mm.Hg.	c.c.			c.c.	mm.Hg.	c.c.		
0·22	7·3	1·01	0·2	2·3	0·17	6·9	1·72	0·1	0·7
0·41	6·1	1·52	0·3	2·3	0·19	6·0	1·93	0·1	0·8
0·30	5·1	1·08	0·3	2·4	0·15	5·2	1·51	0·1	0·9
0·15	4·1	0·58	0·3	2·4	0·20	4·3	2·01	0·1	1·0
0·32	2·3	0·64	0·5	2·5	0·21	2·0	1·04	0·2	1·9
0·21	1·4	0·26	0·8	2·6	0·18	1·1	0·45	0·4	2·1

TABLE III.—*Absorption of Nitrogen by the Sodium-Potassium Alloy when used as the cathode at different temperatures and a pressure of* 4·0 *mm. Hg.*

Volume of N_2 absorbed. (A)	Volume of H_2 liberated in voltameter. (B)	Value A/B	Temperature.
c.c.	c.c.		
0·94	0·41	2·3	50°C.
0·67	0·29	2·3	90°C.
1·08	0·47	2·3	131°C.
0·77	0·34	2·3	200°C.
0·86	0·32	2·7	315°C.
1·29	0·26	4·9	360°C.

7. *Discussion of Results.*

Sodium and potassium absorb nitrogen in the discharge tube and this strengthens the view that the alloy produced by these two metals is a true alloy exhibiting properties similar to its constituents. The fact that it is liquid at ordinary temperatures being a case where the alloy has a lower melting point than either of its constituents. The liquid condition, however, probably accounts for the greater absorption of nitrogen for neither with sodium nor potassium is the absorption as great as with the alloy. As the temperature of the discharge increases the greater absorption in the case of the alloy is due probably to chemical combination, although it is an interesting fact that even at 300°C. no absorption of the gas took place with heat alone, so that the nitrides produced are stable. In all cases of absorption the surface of the metal must be very clean. With amalgams of sodium and potassium there is no absorption, the discharge being carried by the mercury vapour.

ABSTRACT.

The Paper describes a method of showing that absorption of nitrogen takes place in the discharge tube when sodium and potassium are used as the electrodes. The amount of gas absorbed for different quantities of electricity passing in the circuit is measured and is less than that absorbed by the sodium-potassium alloy. It shows also that when the alloy is used the amount of gas absorbed increases with temperature.

DISCUSSION.

Dr. BEATTIE referred to the bearing of the phenomena on photo-electric cells. He asked if it was possible to get such complete absorption as to give an X-ray vacuum, or if there was a dissociation pressure of the gas in equilibrium with the metal. The fact that the alloy absorbed more than either metal separately was possibly a surface tension phenomenon. If a compound was formed of which the surface tension was greater than that of the alloy, it would be drawn inwards, leaving a fresh surface of the uncontaminated alloy in contact with the gas. He had found that when a discharge passed in vacuum between iron electrodes a good deal of hydrogen was emitted. If the discharge was stopped this was re-absorbed, although had the discharge been continued the emission would have gone on as before.

Prof. FORTESCUE said the phenomena was probably the result of several different effects. For instance, if the volatilised films had not been at atmospheric pressure after volatilisation, the fact that they absorbed while the non-volatilised metals did not might be accounted for by the latter having retained gas previously absorbed while exposed to the atmosphere, while the former presented a fresh uncontaminated surface. It was further possible that the thickness of the sodium and potassium layers affected the results. As regards the anode and cathode effects, will this not depend on the relation between the numbers of positive and negative ions present? The author says there is no time effect. Does he mean by this that the absorption will go on, say, for 50 or 60 hours at a constant rate?

Mr. F. J. Harlow pointed out that the films of sodium and potassium were redistilled on silver. Might there not be alloys formed under these circumstances, the alloys and not the metals themselves, possessing the absorptive powers? Had the author used a lime cathode? With this it was possible to get a very steady discharge from the lighting voltage.

Mr. B. P. Dudding thought we were here dealing with problems which Langmuir and others had worked out at some length. Some recent work of Dr. Norman Campbell showed that, with suitable discharge conditions, compounds of almost any two elements could be formed. The essential thing was to remove the products of the reaction.

Mr. Newman, in reply, said that it was not possible to get a very high vacuum with hydrogen. Although it was absorbed by the alloy it was impossible to get rid of the last traces. The surface of the alloy must be very clean. When the discharge is in nitrogen, the surface keeps clean, the scum formed breaking up and going to the sides of the tube, but with hydrogen a black scum forms which requires to be mechanically removed by shaking the tube. As regards the constancy of the rate of absorption, he could not give actual figures, but the rate certainly did not diminish for a very long time. He agreed that it would be difficult to prove that the sodium and potassium films had not alloyed with the silver. He had not tried a lime cathode, but had got very steady discharges by using the type of Wehnelt interrupter, which he had recently shown to the Society.

XX. *A Directional Hot-Wire Anemometer of High Sensitivity, especially Applicable to the Investigation of Low Rates of Flow of Gases. By* J. S. G. Thomas, *M.Sc. (Lond.), B.Sc. (Wales), A.R.C.S., A.I.C.*

Received January 10, 1920.

(Communicated by D. Owen, B.A., D.Sc.)

A type of directional hot-wire anemometer was introduced by the author* for the purpose of readily determining the direction of flow of air or other fluid in pipes or other channels. Essentially the instrument consists of two fine platinum wires arranged parallel and one behind the other in close juxtaposition, transversely to the direction of flow of the gas in the pipe or channel. The wires constitute two arms of a Wheatstone bridge, the remaining arms being formed of a resistance of 1,000 ohms and an arm capable of adjustment. Throughout, a constant current is maintained in the bridge, and the battery terminals are connected to the bridge at the respective ends of the platinum wires, so that the maximum heat is developed in these wires. A galvanometer is inserted in the bridge in the usual manner. A current from 1·0 to 1·5 amperes is suitable for use with the bridge, the wires being thereby heated to a temperature of about 300°C. The operation of the present type of hot-wire anemometer as an indicator of the direction of flow of fluid in the channel wherein it is inserted is dependent upon the fact that of the two fine platinum wires freely exposed to the cooling effect of the current of fluid, that one experiences the greater cooling effect upon which the stream of gas is first incident, this wire exercising by its presence a shielding effect upon the second wire, so that the latter is less cooled by the stream of fluid. A current of fluid being established in the channel, the galvanometer deflection is reversed on reversing the direction of flow of the fluid in the channel. Such a hot-wire anemometer, therefore, affords a ready means of ascertaining the direction of flow of fluids in various units of a complicated network of gas or other mains. Subsequent experience with anemometers of this type has shown that they possess special characteristics which make then particularly useful in the investigation of very low rates of flow. In the region of low velocities, serious difficulties are encountered in the use of hot-wire anemometers

* Journal Soc. of Chem. Ind., 1918.

of the types introduced either by King * or Morris,† owing to the free convection current of fluid ascending from the heated platinum wire. This point it is not proposed to deal with here ; it is treated in detail in the course of an investigation which will appear shortly. Here it is sufficient to state that the disturbing effect of such free convection current on the indications of the instrument is considerably reduced by the use of a second wire shielded from the cooling effect of the stream of fluid, as introduced by Morris to afford temperature compensation in the bridge arms. The effect of the free convection current upon the instrument's indications is, of course, the more completely eliminated, the more nearly the state of the protected wire constituting the compensating arm resembles what may be termed that of the exposed arm of the bridge. The type of directional anemometer described above approaches more nearly this ideal condition of affairs than is the case with Morris's type of instrument. Complete temperature compensation is afforded by the close juxta-position of the wires—in fact, the stability of the zero of the instrument even when used with currents of different magnitudes is a marked characteristic of the instrument. This point is likewise discussed in detail in the work referred to above. The purpose of the present communication is to draw attention to the fact that contrary to expectation, the sensitiveness of the type of directional anemometer described, when subjected to the cooling effect of a slow-moving stream of fluid, is much greater than that employing one freely exposed wire and a second entirely shielded from the cooling effect of the stream of fluid, as embodied in the type introduced by Morris.

Experimental.

For the purpose of the present experiments, an anemometer tube was prepared so that the same wire could be used either as the first exposed wire of a pair constituting a directional anemometer of the type described, or as the exposed wire of a pair, the other member of which was shielded by insertion in a shielding tube. The device is shown in Fig. 1. The directional anemometer is constituted of the fine platinum wires *a* and *b*. The other type of anemometer is constituted of the wires *a* and *c*. The wires *a*, *b* and *c* were inserted at such distances from one another that no appreciable disturbance in the

* "Phil. Trans.," 1914, A. 520, 373-432.
† B.A. Report, September, 1912.

flow was experienced at a owing to the presence of b and c. The mode of insertion of the wires in the tube will be evident from the diagram. Efficient insulation of the wires was afforded by the use of ebonite plugs as shown. c was secured in position by insertion through holes in the copper rods D and E. The wires a and b were secured in slightly different manner as shown, necessitating the provision of strengthening pieces of copper rod driven into the ebonite plugs. The ends of all plugs were carefully shaped, so that no discontinuity in the surface of the tube was produced by their presence. The wires a, b and c were all cut from the same sample of platinum wire,* and were as nearly as possible of the same length, equal

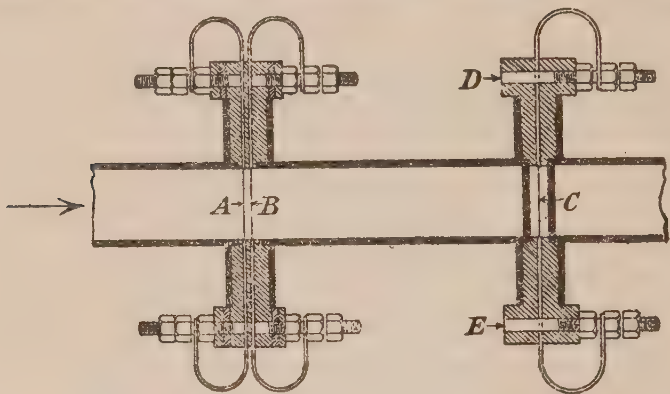

FIG. 1.

to the diameter of the tube, so that they were as nearly as possible of equal resistance. They were attached to the copper rods by means of the minimum amount of silver solder affording a secure junction, and were artificially aged by the passage of a current of 1·5 amperes for two hours. The ends of the tube were connected to 5 ft. lengths of similar tube by means of spigot unions of special design affording a smooth junction. The bridge connections were so made that either b or c could be inserted in the bridge as desired. A voltmeter of resistance 1,000 ohms could be inserted in parallel with a, whereby the resistance of a could be calculated from a knowledge of the current of the bridge, this being maintained constant throughout by means of a rheostat inserted in the battery circuit.

* The platinum wire was of purity 99·5 per cent. A supply of 100 per cent. pure platinum could not be obtained owing to war conditions.

…ctional Anemometer. Current 1·1 *Ampere.*

…ce …nce …).	Drop of potential across first wire (volt.).		Cor. vol. of air per hour (cub. ft.) at 0° and 760 mm.	Equiv. mean velocity in pipe (cm. per sec.)	Resistance of wire (ohm).			
					Zero flow.		With flow.	
With flow.	Zero flow.	With flow.			First wire.	Second wire.	First wire.	Second wire.
982	0·748	0·741	1·52	2·61	0·680	0·687	0·674	0·687
972	0·750	0·734	2·09	4·96	0·682	0·690	0·667	0·688
968	0·750	0·729	2·27	5·39	0·682	0·687	0·663	0·685
958	0·747	0·720	2·40	5·70	0·680	0·686	0·655	0·684
952	0·748	0·715	2·50	5·93	0·680	0·683	0·650	0·683
939	0·750	0·705	2·81	6·67	0·682	0·686	0·641	0·683
935	0·750	0·700	2·90	6·88	0·682	0·685	0·636	0·680
917	0·750	0·689	3·40	8·07	0·682	0·685	0·626	0·683
913	0·750	0·680	3·52	8·36	0·682	0·685	0·618	0·677
Mean	0·749			Mean	0·681	0·685	Mean	0·683

…ectional Anemometer. Current 1·1 *Ampere.*

…ce …nce …s).	Drop of potential across first wire (volt.).		Cor. vol. of air per hour at 0° and 760 mm. (cub. ft.)	Equiv. mean velocity in pipe (cm. per sec.)	Resistance of wire (ohm).			
					Zero flow.		With flow.	
With flow.	Zero flow.	With flow.			First wire.	Second wire.	First wire.	Second wire.
988	0·754	0·750	0·25	0·59	0·685	0·688	0·682	0·690
982	0·754	0·750	0·38	0·90	0·685	0·688	0·682	0·695
975	0·754	0·749	0·52	1·13	0·685	0·688	0·681	0·698
966	0·754	0·750	0·66	1·57	0·685	0·688	0·682	0·706
957	0·752	0·750	0·85	2·02	0·683	0·686	0·682	0·713
944	0·753	0·740	1·15	2·73	0·684	0·687	0·673	0·713
939	0·752	0·741	1·29	3·06	0·683	0·686	0·673	0·717
924	0·754	0·730	1·66	3·94	0·684	0·687	0·664	0·719
922	0·754	0·730	1·73	4·11	0·684	0·687	0·664	0·720
927	0·754	0·711	2·20	5·23	0·684	0·687	0·646	0·697
940	0·754	0·712	2·33	5·53	0·684	0·687	0·647	0·688
963	0·754	0·650	5·17	12·28	0·684	0·687	0·591	0·614
977	0·754	0·592	8·48	20·03	0·684	0·687	0·538	0·551
981	0·754	0·580	10·25	24·32	0·684	0·887	0·527	0·537
981	0·754	0·530	18·60	44·14	0·684	0·687	0·482	0·491
Mean	0·754			Mean	0·684	0·687		

The third arm was adjusted throughout to 1,000 ohms. The bridge being balanced with zero flow of air, the voltmeter was inserted, and drop of potential across *a* measured. A current of dry air was established in the tube. This was derived from a weighted gas holder of 5 cubic ft. capacity, provided with automatic pressure compensation. The air was dried by passage through a column of calcium chloride, its temperature read, and the volume passing down the tube determined by a wet gas meter, by Sugg, of 1/12 cubic ft. capacity, this being standardised by means of the 1/12 cubic ft. bottle prescribed by the Metropolitan Gas Referees. The rate of delivery of the air was extremely steady, the galvanometer deflection not varying by more than one division for any definite rate of flow. Confirmatory readings of the flow of air were taken in all cases. Care was taken that no leakage occurred at the various joints of the apparatus. The drop of potential across *a* was again measured, and the balancing arm adjusted so that the bridge was again balanced. *a* and *c* were now inserted in the bridge, and the readings repeated. A series of similar observations was made for various mean velocities of flow of dry air in the tube. All volumes are reduced to 0°C. and 760 mm. pressure (dry).

Results.

Diameter of tube	2·0534 cm.
Diameter of wire employed	0·0040 in.
Distance between *a* and *b*..................................	0·1 cm.
Distance between *a* and *c*..................................	7·6 cm.

The respective volumes (at 0°C. and 760 mm.) are converted to the equivalent mean velocities of the air stream (reckoned at 0°C. and 760 mm.) by multiplying the former as given in cubic feet per hour by 2·374. The ratio arm in the bridge was maintained throughout equal to 1,000 ohms, and the sensitiveness of the galvanometer was reduced by shunting with 10 ohms, so that a full scale deflection was obtained as the maximum deflection using the directional anemometer. The current employed was 1·1 ampere. The results obtained and calculations based thereon are set out in Tables I. and II., and are represented graphically in Figs. 2 and 3.

Discussion of Results.

From Fig. 2 it is seen that the deflection obtained when using the directional type of anemometer is, except for the extremely low values, strictly proportional to the mean velo-

city of flow until a maximum deflection is reached at b, corresponding to a mean velocity of very approximately 4 cm. per second. Thereafter the deflection diminishes with increasing velocity. It is seen from the form of this portion of the curve that the deflection does not vanish for very high values of the mean velocity of flow in the pipe. The curve OCD is the curve obtained by employing the type of non-directional anemometer, the leading wire being the same as in the directional type. The greater sensitiveness of the directional type is

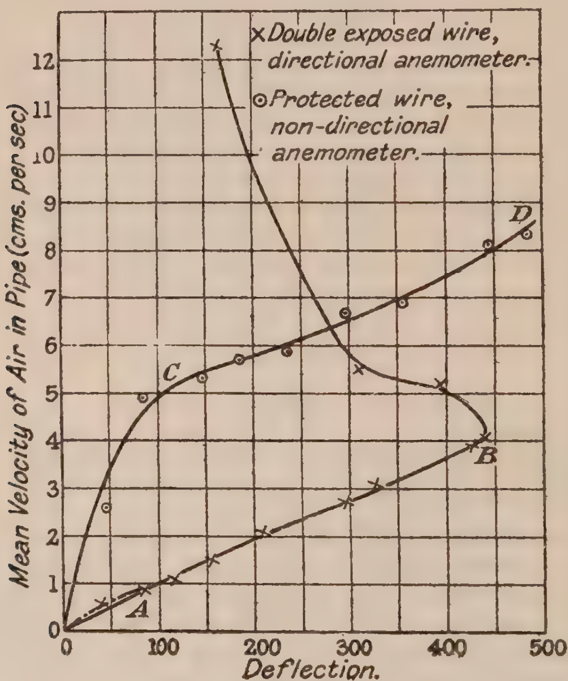

FIG. 2.

clearly indicated. Thus, with the directional type of anemometer, a mean velocity of about 4 cm. per second produces a deflection of 430 scale divisions. The deflection under similar circumstances using the non-directional type is 62. Over the range from A to B the directional type is seen to possess a sensitiveness approximately seven times that of the non-directional type of anemometer. The author proposes to discuss in detail in the Paper referred to above the form

of the curve OCD obtained with the non-directional type of instrument. The form near the origin is determined principally by the magnitude of the free convection current arising from the wire. The almost exact parallelism between the portions AB proceeding from the origin, and CD of the respective curves indicates that, except in the immediate neighbourhood of the origin, the effect of the free convection current is almost completely compensated for by the use of the directional type of instrument. The argument can be developed mathematically, thus : If V_s is the velocity of the stream, and V_c the velocity of the free convection current arising from the wire, the effective velocity of the cooling current of air is $\sqrt{V_s^2+V_c^2}$, with the wire disposed horizontally as in the present experiments. In the case of the wire enclosed in the protecting device, the cooling effect on the wire from its state in an absolutely stagnant medium is entirely due to the free convection current, which we may assume, at least for small values of the impressed velocities of the air stream to be equal to V_c. The differential cooling effect experienced by the wires is therefore that due to a velocity $\sqrt{V_s^2+V_c^2}-V_c$. The author shows in the Paper referred to that in the present case, V_c is of the order 15 cm./sec. Hence, for small values of V_s the differential cooling effect experienced by the two wires is that due to an effective velocity on one wire equal to $V_s^2/2V$, and is thus seen to be determined by both V_s and V_c. This latter is itself dependent on V_s as the temperature of the wire is determined partly thereby. For large values of the impressed velocity of the stream, the differential cooling effect is equal to that due to an impressed velocity $V_s+\frac{V_c^2}{2V_s}-V_c$ and for large values of V_s this is seen to be practically independent of V_c. With the directional type of anemometer, on the other hand, both wires are exposed to the effect of the air stream and their respective free convection currents. The results can best be discussed in terms of the curves shown in Fig. 3, in which the resistances of the respective wires are plotted as ordinates against the impressed velocities of the air stream as abscissæ. Considering first the broken-lined curves C and D representing the variation of the resistance of the leading wire and the protected wire in the non-directional type of anemometer respectively, it is seen that the resistance of the protected wire, and consequently its temperature is,

within the limits of experimental error, constant. The resistance and consequently the temperature of the leading wire falls continuously as the impressed velocity of the air stream is increased. Considering now the curves A and B, which represent respectively the variation of the resistance of the leading and second wires (a and b, Fig. 1) of the directional type of instrument, it is seen that the resistance of the leading wire

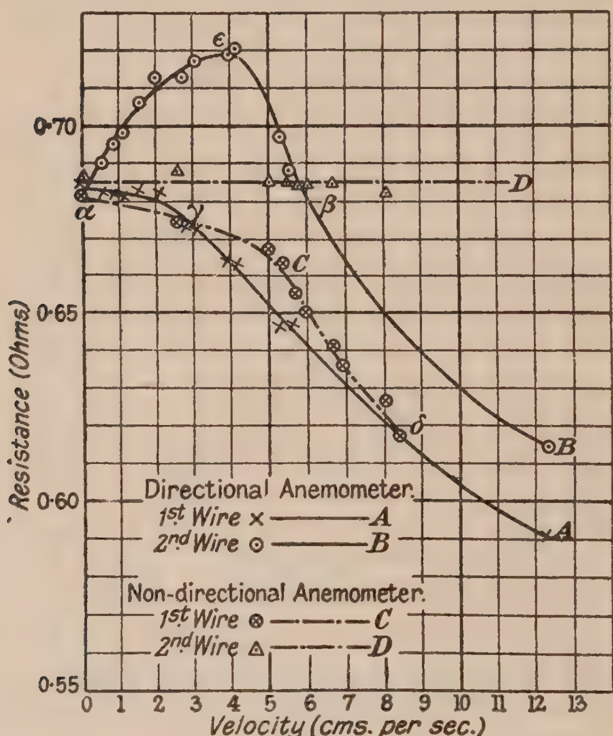

FIG. 3.

(a, Fig. 1) falls continuously with increasing impressed velocity of the air stream, whereas that of the second wire (b Fig. 1) first increases, reaches a maximum at ε, and thereafter diminishes continuously. The greater vertical distances in their initial portions between the curves A and B referring to the wires of the directional anemometer, compared with the distance between C and D in the same region affords a ready

explanation of the greater sensitiveness in the region of low impressed velocities, of the directional type of instrument. The curves A and B have two outstanding features when compared respectively with the curves C and D. These may be characterised thus: The curve C shows that over the region of velocities embraced within $\gamma\delta$, the leading wire a is more cooled when employed in the directional instrument than when used as a component of the non-directional type of instrument. The curve B shows that, contrary to expectations, the second wire b, over the region of impressed velocities, represented by the curve from α to β, far from having its temperature reduced by the stream of air, is considerably warmer when the stream passes over it than when the air is not flowing. These facts are explained as follows: The air reaching b in the stream has already passed over a, and been thereby heated. The radiation loss from b is therefore less when the stream flows than when the medium surrounding b is still. This diminished radiation loss entails a rise in the temperature of b, which is accompanied by an increased loss by free convection from b. The final temperature of b is determined jointly by these two causes. Initially with low impressed velocities, the air carried from a to b is at a comparatively high temperature, and the radiation loss from b is considerably diminished. The rise of temperature of b due thereto is not materially diminished by the accompanying increased loss due to free convection, and the temperature of b, on balance, rises. With increasing impressed velocities, the temperature of the air carried from a to b increases owing to the fact that the time of transit of the air from a to b is diminished with consequent smaller drop of temperature in the air during the transit. Another factor operative in the reverse direction as regards the radiation from b requires attention. With increasing impressed velocity, the actual temperature of the air leaving a diminishes continually. The actual temperature of the air in the stream in the neighbourhood of b is therefore determined by (1) the velocity of the stream and (2) by the time of transit which determines the loss during transit. Initially with low velocities, the actual temperature of the air in transit is such that on balance, the effect of the high temperature due to the small motion is increasingly predominant over the subsequent drop in transit from a to b. A point is therefore attained with increasing impressed velocity such that the temperature of b attains its maximum value. Thereafter, the impressed velo-

city is such that the counter effects of initial heating of the air at a and subsequent cooling during transit from a to b operate so as to reduce the temperature of b owing to the growing predominance of the latter effect. When such an impressed velocity is attained that the heat acquired by passage over a is subsequently exactly that lost during the time of transit from a to b, it is clear that the temperature of b is exactly what it would be in the absence of any impressed velocity. This condition is represented by the point β of the curve B. Thereafter with increasing impressed velocity of the stream, the temperature of b is continuously reduced by the stream, as shown by the portion of the curve βB. Considering now the portion of the curves A and C, it is seen that the temperature of the wire a over this region of impressed velocities is actually *lower* when the adjacent wire b is heated than when b is not heated. This phenomenon is explained as follows: When a and b are both heated, and an impressed stream of air passed over them, the cooling effect experienced by them is the resultant effect of the impressed velocity and the free convection currents arising from the wire. It is readily seen that the resultant current of air from the wire b, which originally rises vertically upwards, therefrom when the impressed velocity of the air stream is zero, tends towards the horizontal as the impressed velocity is increased. This free convection current from b produces a cooling convection current in the neighbourhood of a, so that when b is heated by the electric current, the cooling due to this induced convection current in the neighbourhood of a more than counterbalances the heating of a due to radiation from the heated wire b. The temperature of a is, under these circumstances, actually reduced below what it would be if b were not heated.

It should now be clear that the extreme sensitiveness of the directional type of anemometer, at low velocities, arises partly from the increased temperature of the second wire due to causes originating in the first wire of the pair, and partly to the diminished temperature of the first wire originating in the second wire.

The directional type of anemometer, with its almost perfect compensation of free convection effects is of especial application in the investigation of low velocities of flow. A still more sensitive arrangement than that described is afforded by utilising two couples of wires in close juxta-position, the couples being separated by an interval and constituting there-

from a Wheatstone bridge. Thereby the sensitiveness is increased a further two-fold.

The work detailed herein was carried out in the physical laboratory of the South Metropolitan Gas Company, and the author desires to express his thanks to Dr. Charles Carpenter for the ready provision of the necessary facilities, and for permission to publish the results obtained.

ABSTRACT.

The instrument consists of two fine platinum wires mounted close together, and forming two of the arms of a Wheatstone bridge. These are heated by the current in the bridge. When a stream of gas moves in a direction perpendicular to the wires but parallel to the plane containing them, the leading wire is cooled, while the second wire, being shielded by the first, is not cooled as much, and may actually be heated on account of the air flowing past it being warmed by the first wire. A deflection of the galvanometer is obtained, therefore, which is reversed if the flow of gas is in the reverse direction. For low rates of flow of gas, the instrument is much more sensitive than the non-directional hot-wire anemometer. An explanation of the increased sensitivity is advanced.

DISCUSSION.

Dr. Ezer Griffites said he had been working with hot-wire anemometers recently, but they required somewhat complicated gear. It is assumed in working out the theory that the flow of gas is parallel to the axis of the tube. What happens if this is not so owing to the proximity of bends?

Dr. Tucker said he had used a hot-wire anemometer for impulsive and alternating flows. But the hot-wire inevitably produces its own direct current due to convection. He had found the curious result that the sensitivity for alternating currents could be increased many times by increasing the strength of the direct current to which the wire was exposed. Had the author attempted to increase the sensitivity by reducing the diameter of the wires? The sensitivity rose very rapidly as the diameter was reduced.

Dr. E. H. Rayner asked if experiments had been made with different dimensions to see if the range of usefulness could be increased. He thought the velocities in gas mains would usually be much greater than 4 cm. per second, which seemed to be about the maximum useful range with the present instrument.

Mr. F. E. Smith said he was not convinced by the author's explanation of the first wire being further cooled, due to the proximity of the second hotter wire. He suggested a simple experiment to prove whether or not this cooling actually took place.

Dr. Borns asked if all the experiments had been in coal gas.

Prof. Rankine asked if any difficulties arose due to the wires not remaining in the plane parallel to the flow. One would expect sagging to occur, and the two wires would not necessarily bulge in the same direction.

Dr. D. Owen suggested that there might be some advantage in this respect if grids were used instead of wires.

Mr. J. Guild asked as to the variation in the readings of the instrument due to alteration of the inclination of the plane of the wires to the stream of gas. To within what limits could the direction of flow be determined?

Dr. L. Hopwood said the author was to be congratulated in finding a directional instrument which was more sensitive than a non-directional one. Direct measurements could be made on the convection current effects by using a vertical flow. In one direction the convection currents would assist the flow, and in the other would be against it.

Dr. T. Barratt (communicated): I have read Mr. Thomas's interesting Paper with much pleasure. His explanations of the more or less intricate phenomena involved are convincing and simple. In one particular, however, the facts can be explained more simply as well as more accurately. Radiation from thin wires, even at moderately high temperatures, forms a very small percentage of the total heat lost. In the case of Mr. Thomas's anemometer wires, this percentage is probably less than 5, and certainly less than 10 if the wire is "bare"—*i.e.*, slightly soiled by exposure to air, &c. That is to say, nearly all the lost is lost by convection. An explanation of the observed facts can be made, however, without any assumption of radiation at all. Assuming that nearly all the loss of heat is due to convection, "free" or "forced," the air reaching b from the direction of a is considerably warmer than the rest of the gas. Hence the convection from b (which strictly obeys Newton's first power law) is considerably reduced, resulting in a higher temperature for the wire b. At higher velocities of the cooling fluid the air from a reaching b is not so warm, so that b is cooled more than at low velocities. The remaining explanations can follow on the same line of reasoning.

The Author, in reply to Dr. Griffiths, stated that as regards disturbed stream lines, King states ("Phil. Trans.," A.520, 1914, p. 407) that the experiments of A. M. Gray show that the hot-wire anemometer renders possible a consistent measure of turbulent flow where other methods of measurement would be inadequate. King ("Phil. Trans.," A.520, p. 425) has studied the effect of inclination of the hot wire to the stream on the convection constants of small wires. With the directional type of instrument, the maximum shielding of the second wire is obtained when the wires are normal to the gas stream. The point has been investigated by the author, and the variation of galvanometer deflection with variation of the plane of the wires to the free convection current arising from the wires has been made the basis of the construction of an electrical inclinometer. The variation of sensitivity with diameter of the wire as mentioned by Dr. Tucker in the case of the non-directional instrument has been carefully investigated by King ("Phil. Trans.," *loc. cit.*, p. 383 *et seq.*). In the case of the directional instrument, the increased sensitivity due to the use of a given wire is somewhat reduced by the smaller shielding effect they afford. In reply to Dr. Rayner, the directional type of instrument is not intended to be used as a measurer of gas flow, save in its special sphere of sensitiveness, viz., for low rates of flow of gas. Certain operations in gas works are carried out in the mains only when the gas flows in one direction. This direction being indicated, the actual velocity in the main can, if need be, be ascertained by a non-directional type of instrument. The author's experiments were not confined to coal gas, but embraced experiments using currents of air, carbon di-oxide, oxygen, hydrogen and other gases. Difficulties are encountered when using a bare wire anemometer with gases such as hydrogen or methane, &c., particularly if the wires are used at an elevated temperature. These are entirely eliminated if the wire is coated with glass. The instrument is then considerably more robust and its sensitivity little reduced. The experiment suggested by Mr. Smith had been tried and the cooling effect confirmed. The existence of the cooling effect seemed to depend upon the relative disposition of the wires as regards distance apart. The sagging referred to by Dr. Rankine did occur, but its influence could, with care, be

eliminated. This was done by ensuring that the wires, initially in a diametral longitudinal section of the pipe, remained so when heated. The necessary adjustment is easily effected by rotating the rods to which the wires are affixed. In any case, the sagging can be reduced by employing a small heating current, and can be eliminated entirely by employing a spring-fork support for the wires as introduced by King. The author agreed with Dr. Owen that the use of a grid of parallel wires would reduce the variation due to sagging. Experiments on the lines suggested by Dr. Hopwood have already been carried out by the author, and direct measurements made of the magnitude of the velocity of the free convection current arising from fine heated platinum wires. The results obtained enable the temperature coefficient of the free convection velocity to be very accurately determined. Mr. Guild's query is answered in the reply to Dr. Griffiths. The author is unable to assent to the explanation of the phenomena suggested by Dr. Barratt. Dr. Barratt's experiment on the radiation and convection losses from a heated wire have extended only up to a temperature (100°C.) considerably below those employed in the present case. It is interesting to note that he finds (" Proc." Phys. Soc., 1915, 28, p. 10) that the proportion of radiation is greater at higher temperatures. In the absence of data, it can hardly be claimed that omission of all consideration of radiation, even though this amount to only 5 per cent. of the total heat loss involved, can afford an accurate interpretation of the results. In the first place, it is not the actual magnitudes of the respective losses by radiation and convection that are involved, but rather the variations of these with variation of the temperature difference between the wire and its surroundings under the peculiar conditions of temperature distribution around the wire in the present experiments. In this connection it must be noted that the *resultant* direction of flow of the hot stream of gas from the first wire is such that the distribution of temperature round the second wire due thereto is asymmetric, and with low rates of flow of gas the temperature difference between the medium above the wire and that below is increased on this account, the hot current of gas from the first wire being directed towards the region above the second wire, while the convection current in the immediate vicinity of the second wire is little disturbed, owing to the shielding influence of the first wire. There is, therefore, an increased convection current from below upwards across the second wire, with a consequent *increased* convection loss of heat, and not a *decreased* loss, as suggested by Dr. Barratt.

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